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Nr 22 1948

THEORY AND APPLICATIONS OF TROCHOTRONS

BY

H. ALFVÉN, L. LINDBERG, K. G. MALMFORS,
T. WALLMARK AND E. ÅSTRÖM

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Stockholm, March 8th, 1949.

Professor H. Alfvén
The Royal Institute of Technology
Stockholm 26, Sweden

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Mr. William E. Barbour, Jr.
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Dear Mr. Barbour,

Best thanks for your letter of January 21st. It may be of interest to you that the Ericson Company has a non exclusive license in U.S.A. but only for their own use with no right to sell to other companies. We are looking for a co-operation with an American company for manufacturing the trochotrons and the possibilities to sell tubes to you depend upon the agreements, which can be made with such a company. We have so far not made any decisions, and if you have any suggestions, we are interested to hear from you. I send under special cover a publication "Theory and applications of trochotrons", which represents the state of our investigations about one year ago. Since then we have made very good progress. We have now tubes, which work satisfactorily with 5 mA at 200 volts or 2 mA at 100 volts. We have also made some new types of tubes, which give as much as 10 mA at a maximum voltage of only 40 volts, but this type is so far not investigated very carefully.

Very truly yours,
H. Alfvén
H. Alfvén

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T. WALLMARK AND E. ÅSTRÖM

GÖTEBORG 1948
ELANDERS BOKTRYCKERI AKTIEBOLAG

ON TROCHOIDAL ELECTRONIC BEAMS AND THEIR USE IN ELECTRONIC TUBES ("TROCHOTRONS")

BY

HANNES ALFVÉN

1. Introduction

In a homogeneous magnetic field a charged particle with a velocity perpendicular to the magnetic field moves in a circle. If the magnetic field is slightly inhomogeneous, or if it is superimposed by a crossed electric field, the circle »drifts», so that the path of the charged particle becomes a trochoid. If the particle has a velocity component parallel to the magnetic field, a translational motion is also produced. Although this has been well known for some time, the trochoidal motion of electrons has only been employed in a few special cases. The first to make use of the trochoidal motion seems to have been THIBAUD,[1]) who analyzed β-rays from radioactive substances by means of an inhomogeneous magnetic field, in which the β-rays moved along trochoidal paths. The trochoidal motion in a magnetic field superimposed by an electric field is discussed, in principle, in many textbooks (especially in connection with the theory of magnetrons), but a more detailed analysis of the properties of the motion seems to be lacking.

In cosmical physics, charged particles in the magnetic fields of the earth, sun, or other celestial bodies, usually move in orbits of trochoidal character. For the study of charged particles in cosmical physics, the investigation of the properties of the trochoidal motion

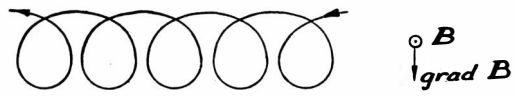

Fig. 1. Motion in inhomogeneous magnetic field.

[1]) THIBAUD, Cim. *15*, 313, 1938.

is of paramount interest. When trying to check the results by laboratory experiments, it was found in this laboratory some years ago that trochoidal beams of electrons possessed properties which make them very interesting, even for technical purposes. Investigations were started in order to study trochoidal beams, and to construct tubes in which use was made of their properties. These tubes have been called trochotrons.

A preliminary report on the trochotron was published some time ago by H. ROMANUS and the present author.[1]) Others who have taken part in the investigations of trochoidal beams and the technical development of the trochotron, are J. BJÖRKMAN, G. HAMBRAEUS, K. G. HERNQVIST, L. LINDBERG, S. LUNDQUIST, K. G. MALMFORS, B. STRÖMBERG, T. WALLMARK, S. WARRING and E. ÅSTRÖM. Their results are published in the following papers by WALLMARK,[2]) LINDBERG,[3]) ÅSTRÖM[4]) and MALMFORS.[5])

The purpose of the present paper is to give a general survey of the properties of trochoidal electronic beams, and their application in electronic tubes.

2. On the Trochoidal Motion of Charged particles

Suppose that a charged particle moves in a magnetic field, eventually superimposed by an electric field, the inhomogeneities of which are so small that the relative change in field strengths over the radius of curvature r of the particle path is small. Then the motion may be approximated by a circular motion, around a centre which drifts. The radius of curvature is given by

$$r = \frac{1}{eB}\sqrt{2\,m\,W_{\perp}} \tag{1}$$

where B is the magnetic field, e and m the electronic charge and mass, and

$$W_{\perp} = \frac{1}{2}\,m\,v_{\perp}^2 \tag{2}$$

[1]) ALFVÉN, H.: and ROMANUS, H. Valve with trochoidal electronic motion, Nature, *160*, p. 614, 1947.

[2]) WALLMARK, T.: Design and properties of trochotrons.

[3]) LINDBERG, L.: Design of trochotron circuits.

[4]) ÅSTRÖM, E.: Experimental investigations on an electronic gas in a magnetic field.

[5]) MALMFORS, K. G.: On the instability of an electron gas in a magnetic field.

the kinetic energy associated with the circular motion $v_\perp$. M. K. S. A. units are used. The inhomogeneity of the magnetic field, and the eventually applied electric field, make the centre of the circle drift with a velocity u with the components $u_{||}$ parallel to and $u_\perp$ perpendicular to the magnetic field. These components can be computed from the differential equations[1])

$$u_\perp = -\frac{1}{e\,B^2} \cdot \vec{B} \times (\vec{f} + \vec{f^m} + \vec{f^i}) \tag{3}$$

$$\vec{B}\,(\vec{f} + \vec{f^m} + \vec{f^i}) = 0 \tag{4}$$

In these equations f is the force acting upon the particle. If this is due to an electric field we have

$$\vec{f} = e\vec{E} \tag{5}$$

The fictive force

$$\vec{f^m} = \mu \text{ grad } B \tag{6}$$

with

$$\mu = \frac{W_\perp}{B} \tag{7}$$

accounts for the drift due to the inhomogeneity of the magnetic field. Further

$$\vec{f^i} = -m\frac{\vec{du}}{dt} \tag{8}$$

has the character of an inertia force. During the motion, μ remains constant.

We shall here confine ourselves to the case of a homogeneous magnetic field, which, in fact, is must important in this connection, because most of the work on trochoidal beams has been carried out under these conditions. Then we have $f^m = 0$. Denoting the

[1]) ALFVÉN, H.: On the motion of a charged particle in a magnetic field, Ark. f. mat. astr. o. fys. *27 A* No. 22.

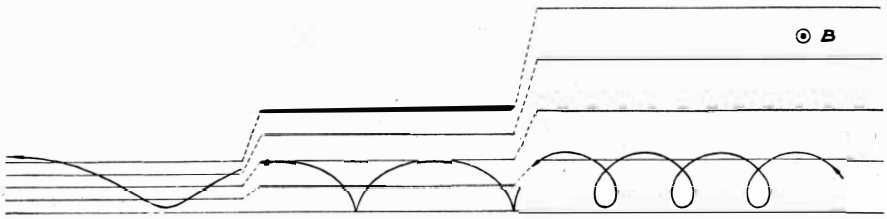

Fig. 2. Motion in homogeneous magnetic field and crossed electrical field. Left $u_\perp = 2\,v_\perp$; centre $u_\perp = v_\perp$; right $u_\perp = \frac{1}{2}\,v_\perp$. Horisontal lines are equipotential lines.

components of E parallel and perpendicular to B by $E_{||}$ and $E_\perp$ and inserting (5) and (8) into (4) we obtain

$$m\,\frac{du_{||}}{dt} = e\,E_{||} \tag{9}$$

expressing the simple fact that the motion $u_{||}$ parallel to B is an ordinary accelerated motion, unaffected by the magnetic field. From (3) we obtain in a similar way

$$\vec{u}_\perp = -\frac{1}{B^2}\cdot\vec{B}\times\left(\vec{E} - \frac{m}{e}\,\frac{\vec{du}}{dt}\right) \tag{10}$$

When the rate of change in the drift velocity is slow — which is often the case — the inertia term may be neglected and we simply have

$$\vec{u}_\perp = \frac{1}{B^2}\,\vec{E}\times\vec{B} \tag{11}$$

Confining ourselves to the two-dimensional case ($E_{||} = 0$; $v_{||} = 0$) it is obvious from (11) that the drift is perpendicular to the electric field. As the field is perpendicular to the equipotential surfaces (in the two-dimensional case: equipotential lines), the result is that (for $f^i = 0$) *the drift follows an equipotential line*. The curve described by a point which moves on a circle, the centre of which drifts, is called a *trochoid* (or in the special case when the circular velocity equals the drift velocity, a *cycloid*).

The velocity of the particle at a certain point is

$$\vec{v'} = \vec{v_\perp} + \vec{u_\perp} + \vec{u_{||}} \tag{12}$$

and consequently the kinetic energy

$$W' = \frac{1}{2} m v'^2 = W_\perp + W_{||} + \frac{1}{2} m u_\perp^2 + \vec{mv_\perp} \cdot \vec{u_\perp} \tag{13}$$

where

$$W_{||} = \frac{1}{2} m u_{||}^2 \tag{14}$$

For motion perpendicular to B, $W_{||}$ is zero, and the kinetic energy varies between the limits [compare (2)]

$$W'_{\max} = W_\perp (1 + \eta)^2 \tag{15}$$

and

$$W'_{\min} = W_\perp (1 - \eta)^2 \tag{16}$$

with

$$\eta = |u_\perp| / |v_\perp| \tag{17}$$

The average is

$$W' = W_\perp (1 + \eta^2) \tag{18}$$

Since the sum of kinetic energy and potential energy remains constant, it is easy to find the corresponding electrostatic potentials. In fact, supposing that electrons are emitted from a cathode at zero potential, and that the circular motion represents the energy $W_\perp = e V_o$, the centre moves at the potential

$$V = V_o (1 + \eta^2) \tag{19}$$

The trochoid is confined to the region between the potentials

$$V_1 = V_o (1 - \eta)^2 \tag{20}$$

and

$$V_2 = V_o (1 + \eta)^2 \tag{21}$$

As long $u_\perp$, and consequently η, remains constant, the drift takes place along an equipotential line. A change in $u_\perp$ means that the inertia term in (3) and (10) differs from zero, and the path of the centre is displaced from one equipotential line to another. In cases of most interest in the following, the deviation of the centre path from an equipotential line is not very large. In fact, if $u_\perp$ varies from 0 (corresponding to a stationary circle) to $v_\perp$ (corresponding to cycloid case), the centre line is displaced perpendicular to the equipotential lines, only half way to the upper limit of the trochoid, i. e. a quarter of the breadth of the trochoid.

3. Properties of a trochoidal beam

A trochoidal beam is formed, for example if electrons are shot out with a certain initial velocity v_i in a magnetic field B, crossed by an electric field E. Let us suppose that the electric as well as the magnetic field is homogeneous, and that the initial velocity is perpendicular to the magnetic field, but makes the angle φ with the electric field. At the point of emission, the vector sum of the drift $\vec{u}_\perp$ and the circular velocity $\vec{v}_\perp$ should equal $\vec{v}_i$, which gives

$$v_\perp^2 = (v_i \sin \varphi - u_\perp)^2 + (v_i \cos \varphi)^2 = v^2 + u_\perp^2 - 2\, v_i\, u_\perp \sin \varphi \quad (1)$$

where $u_\perp = \mathrm{E/B}$. Introducing

$$V_B = \frac{m}{2\,e}\left(\frac{E}{B}\right)^2 \quad (2)$$

$$V_i = \frac{m}{2\,e}\, v_i^2 \quad (3)$$

and observing that $V_o = \frac{m}{2\,e}\, v_\perp^2$, we find

$$V_o = V_i + V_B - 2\sqrt{V_i\, V_B}\, \sin \varphi \quad (4)$$

Introducing this value into (2.19), (2.20), and (2.21), we find the average, minimum and maximum voltage of the trochoid. The value of η is given by

$$\eta = \sqrt{V_B/V_o} \quad (5)$$

Let us discuss three cases of special interest:

1) The electrons are emitted with no initial velocity (Fig. 3 a). Then we have $V_i = 0$; $V_o = V_B$; $\eta = 1$. The centre of the circle drifts along the equipotential line

$$V = 2\, V_B \tag{6}$$

An electrode may be hit by the beam if its voltage is between

$$V_1 = 0 \tag{7}$$

and

$$V_2 = 4\, V_B \tag{8}$$

The quantity V_B is given by the ratio of the electric and magnetic field according to (2). As a numerical example, if E = 400 volt/cm and B = 200 gauss, we have $V_B = 11$ volt.

In this case the path is a cycloid.

2) The electrons are shot out anti-parallel to the electric field by an electron gun, which, consequently, is directed towards the positive electrode (Fig. 3 b). The cathode of the gun is at zero, the anode at V_i, which at the same time is the voltage at the point where the gun is situated. The electric field is still homogeneous. We put

$$\varkappa = V_i / V_B \tag{9}$$

As $\varphi = \pi$ we obtain

$$V_o = V_B\,(1 + \varkappa) \tag{10}$$

and as $\eta = (1 + \varkappa)^{-\frac{1}{2}}$ the maximum and minimum voltages are

$$V_1 = (2 + \varkappa - 2\sqrt{1 + \varkappa}) \cdot V_B$$

$$V_2 = (2 + \varkappa + 2\sqrt{1 + \varkappa}) \cdot V_B$$

3) The electrons are emitted uniformly, in all directions of the plane, with an initial velocity corresponding to V_i. This case is realized approximately if the cathode is a thin wire parallel to B, and the voltage of the wire differs from the voltage at the same point of the undisturbed field by V_i. (Fig. 3 c).

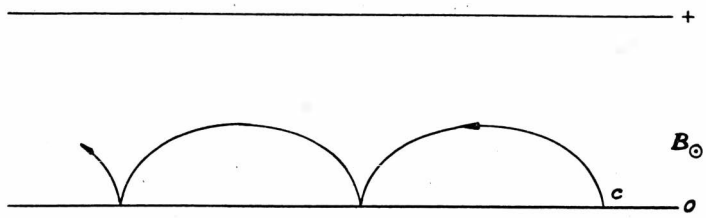

Fig. 3 a. Electrons emitted from cathode c with no initial velocity.

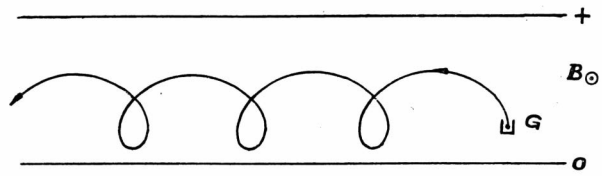

Fig. 3 b. Electrons emitted from electron gun G with initial velocity.

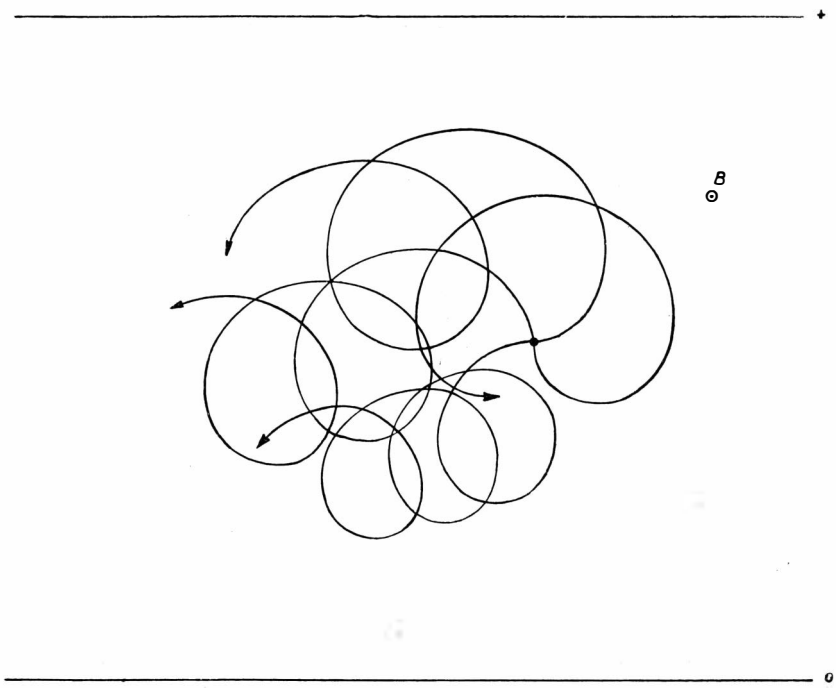

Fig. 3 c. Electrons emitted from a filament at the voltage of the negative plate.

A complex trochoidal beam is formed, consisting of a continuum of trochoidal beams with potentials V_o varying between the limits V_o' and V_o''.

Confining ourselves to the case $V_i > V_B$, which is of most interest, we have

$$V_o' = V_i + V_B - 2\sqrt{V_i V_B} = V_B(\sqrt{\varkappa} - 1)^2$$

$$V_o'' = V_i + V_B + 2\sqrt{V_i V_B} = V_B(\sqrt{\varkappa} + 1)^2$$

where $\varkappa = V_i/V_B$. Introducing (5) we find from (2.19) and (2.21) that an electrode may be hit by at least a part of the complex trochoidal beam if its voltage is above

$$V_1' = V_o'(1 - \eta)^2 = V_B(\sqrt{\varkappa} - 2)^2$$

or below

$$V_o'' = V_o''(1 + \eta)^2 = V_B(\sqrt{\varkappa} + 2)^2$$

For $\varkappa = 5$ we have $V_1' = 0.06\ V_B = 0.01\ V_i$; $V_2'' = 18.0\ V_B = 3.6\ V_i$.

4. Current-voltage characteristic of an electrode

For the discussion in § 3, we have assumed the electric field to be homogeneous. This means that the electrode, which receives the trochoidal beam, or part of it, must not disturb the field. Hence, when we change its voltage, we must at the same time displace it in the field. According to the assumption that the receiver is a plane sheet, parallel to the equipotential surfaces, and with its breadth at least equal to the periodic length of the trochoid, we obtain the current-voltage characteristics of Fig. 4 for the three cases discussed

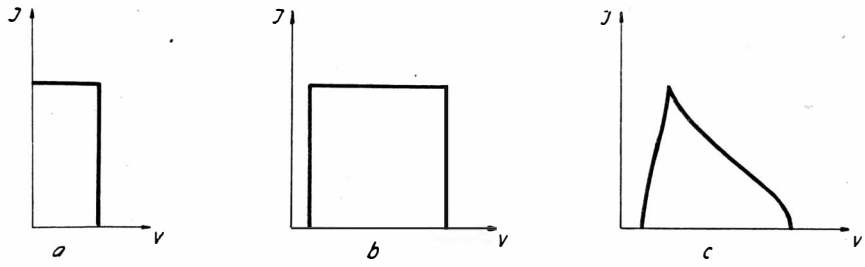

Fig. 4. Theoretical current-voltage characteristics for the three different electrode arrangements of fig. 3.

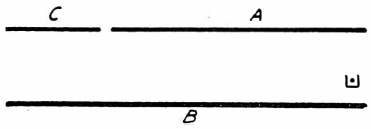

Fig. 5.

in § 3. The last one is obtained by numerical integration according to the assumption mentioned above.

In practice we wish to keep the electrode configuration unchanged, and only vary the voltage of the receiver electrode. Then the electric field becomes, in general, inhomogeneous, and the drift changes. The effect of this may be estimated from § 2. For example, suppose that the electrode configuration is as shown in Fig. 5. A trochoidal beam is emitted from an electron gun, placed between two parallel plates A and B, and received by a plate C which is the continuation of the positive plate A. The highest voltage at which the plate C receives electrons is

$$V = V_o (1 + \eta)^2$$

If the electrode distance is a we have

$$\eta = \frac{u_\perp}{v_\perp} = \frac{1}{v_\perp} \frac{E}{B} = \frac{V}{a}$$

with

$$a = a v_\perp B$$

Thus, the high energy cut-off is given by

$$V = V_o (1 + V/a)^2$$

The low energy cut-off is more difficult to compute, because the beam passes between A and C, and the sharp edges introduce inhomogenities, so that the electric field varies considerably over the distance r. This means that our formulae are no longer valid. The path must be found by numerical or graphical integration.

In the most important practical cases the characteristic curves are not very different from the ideal curves of Fig. 3.

In case 1) in § 3, the cathode was supposed to be a thin filament parallel to B. The emitted electrons move in a cycloid. Suppose that instead, the cathode is a plane surface perpendicular to E, with its breadth equal to the distance between two consecutive cusps of the cycloid. If the emission of the cathode is uniform we obtain

a continuous and uniform distribution of cycloids, all having the same potential. We call such a beam a *trochoidal beam without ray structure*, whereas if some of the cycloid paths have higher electronic density than others, we speak of a beam with more or less pronounced ray structure.

When the beam is emitted by an electron gun, a beam with ray structure is obtained. If the electrons leave the gun under a large solid angle, the ray structure is usually smoothed out considerably after a few loops. In case 3) of § 3, the ray structure is usually not very pronounced. In general, space charge, especially near the cathode, tends to smooth out the ray structure.

In particular when the receiving electrode is of small dimensions or has sharp edges, the ray structure of the beam makes the current-voltage curves lose their smoothness, and introduces a multitude of secondary maxima and minima.

5. General properties of a trochotron

The principle discussed above can be used in a great variety of ways to construct tubes for different purposes. During the last few years more than a hundred different tubes have been constructed and tested in this laboratory. In order to illustrate some important properties of these tubes, we shall briefly discuss one of the simple trochotron types. A more detailed treatment will be found in the paper by WALLMARK. The construction of a simple trochotron is seen from Fig. 6. A magnetic field of the order of a few hundred gauss is perpendicular to the plane of the paper. The cathode c may be a filament or a small electron gun. The electrode r, called the »rail»

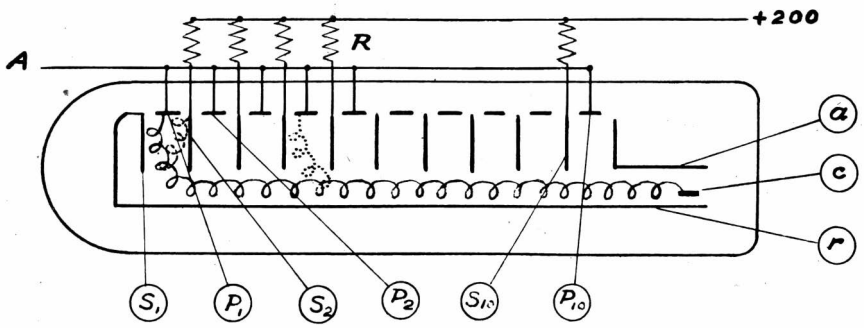

Fig. 6. Simple trochotron circuit.

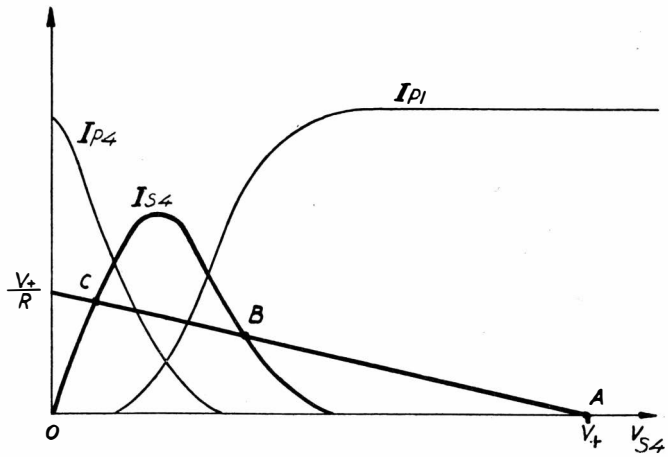

Fig. 7. Current-voltage curve of trochotron.

because the trochoid rolls upon it, is at zero, or a few volts negative in relation to the cathode. The electric field is produced between the rail and a series of positive electrodes, of which the »anode» a is close to the cathode. The other positive electrodes are of two different types: the long spade-like electrodes s_2 s_3 . . . s_{10} are called »spades» and the small electrodes p_1 p_2 p_3 . . . p_{10} are called »plates». All these electrodes are, normally, at a positive voltage of about 200 volts, except spade 1, which is connected to the rail. The trochoidal beam, emitted from the cathode, follows approximately an equipotential line, which runs between the rail and the spades through the entire tube. At the end of the tube it passes through the small spacing between the first spade and first plate. When the trochoidal beam reaches this space the electrons hit plate 1. It has been found that practically the whole current emitted from the cathode (more than 99.9 %) can reach plate 1, in spite of the fact that it passes close to all the positive spades on its long journey.

If the voltage of one of the spades, say s_4, is changed, we obtain the current-voltage characteristic as shown in Fig. 7. The current to the plates p_1 and p_4 is indicated in the same figure. Hence, the spade acts as a switch, which makes the current go alternately to p_1 or p_4. If the spade s_4 is connected to the high voltage V_+ by a sufficiently large resistance R, the switch is made »self-locking». In fact the current-voltage characteristic of s_4 is cut by the straight line

$$V = V_+ - Ri$$

at three points A, B and C, which represent the possible states. Of these points, B is unstable, so that in reality there are two stable positions, A and C. If the »switch» is in position A, the spade takes no current and is consequently at constant high voltage. The beam goes to plate p_1. At point C the spade takes so much current that the voltage drop in the resistance keeps it at a low voltage. That part of the beam, which is not taken by the spade, goes to plate p_4.

Hence it is seen, that in a trochotron every spade may be used as a self-locking switch, which may be operated by external pulses. The pulse technic is discussed in detail in the paper by LINDBERG. Only a simple example will be given here.

Suppose that all spades are connected by resistances R to the high voltage, and that all plates are connected together to an input terminal A. The plates are initially at a high voltage, and the beam lands on plate p_1. If a negative pulse is applied to the input terminal, the voltage of all the plates drops. The equipotential line, which initially passes through the specing between the spade s_1 and plate p_1, is now changed, so that it passes between plate p_1 and spade s_2. Hence the current goes to spade s_2, the voltage of which drops, because of the series resistance. The result is that the beam is switched over from plate p_1 to plate p_2. If the negative input pulse has passed, when the switching is finished, the beam remains in the second box as long as no other pulse is applied. A second pulse switches it over in the same way to the third box, so that the current flows to spade s_3 and plate p_3. Every new pulse is effective only in the box, where the beam terminates, because the plates to the left or to the right of that box are too far away from the trochoidal beam to affect it. When finally the beam has been brought to the last box (spade s_{10} and plate p_{10}), the next negative pulse brings it over to the anode a. If this is connected to a series resistance, its voltage drops, so that the emission from the cathode is cut off. Then all voltages in the trochotron are restored to the initial conditions, so that when the anode voltage goes up again, the beam lands on plate p_1.

The result is that the trochotron in this circuit acts as a *cyclic switch*. If the trochotron contains ten boxes, it may be used as »scale-of-ten», so that every tenth pulse applied to the input terminal, produces an output pulse, which may be tapped for example from the anode or from a spade.

In order to make the tube independent of the shape and length

of the input pulses, the circuit must be made a little more complicated, as is discussed in LINDBERG's paper.

If all the plates are brought down to a constant low voltage, the beam will circulate in the trochotron by itself, and give a continuous series of pulses. The spacing between adjacent pulses can be adjusted in an arbitrary way, for example by connecting condensers across the spade resistors R. Even if the terminal A is put at a high voltage, the device may be self-switching, if large resistances are connected in series with the plates. The spacing between two consecutive pulses may be changed by changing the voltage of the corresponding plate, and as shown in LINDBERG's paper, this offers the possibility of using a trochotron as a time-pulse modulator.

The same tube may be used as a switch, also in the following way. All the plates and spades are connected to the successively rising steps of a voltage divider. The beam goes to the electrode, which has the voltage characterizing the beam. If this is altered by changing the voltage of the electron gun, the beam is switched over from one electrode to another.

These examples have been given in order to show some of the possibilities of using the trochoidal beams. Investigations in this laboratory have revealed many other applications, some of which are described in the cited papers. The field seems to be rich, and probably many other applications are possible.

6. On Electrons in a Magnetic Field

When studying the properties of the trochotrons a remarkable phenomenon has been found. As soon as the current in the tube exceeds a certain limit (of the order of 1 mA), current starts to electrodes, which are negative in relation to the cathode.

The phenomenon can be studied in trochotrons of the type discribed in § 5, but also on much simpler devices, for example in the case when the trochotron consists of a positive and a negative plate, a cathode c, and a receiver plate r, as shown in Fig. 8. In the direction of the magnetic field the trochotron is limited by two side plates. If the negative plate and the side plates are a few volts negative in relation to the cathode, the positive plate about 200 volts positive, and the receiver plate about 100 volts positive,

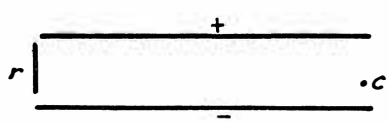

Fig. 8. —

the main current at low emission goes to the receiver plate. When the emission goes up to the order of one or a few mA, a large part (in certain cases more than 50 %) of the emission goes to the negative plate and side plates. The positive plate receives only very little current, the rest going to the receiver plate. One might at first be inclined to attribute such a phenomenon to oscillations, especially since the negative characteristics of a trochotron make it easy to produce oscillations. The phenomenon occurs, however, even if there is definitely no alternating voltage on the electrodes. Hence it is necessary to assume that by some mechanism some of the electrons gain sufficient energy to reach the negative electrode. As ÅSTRÖM has shown, the occurence of »negative current», as the phenomenon may be called, is always accompanied by a noise, which is more or less uniformly distributed over all frequencies, from zero up to above the gyro frequency. Hence an electric field of noise character is probably produced by electrons moving in the magnetic field. The presence of an electric field in the trochotron is of no essential importance in this connection. If we regard the electrons in a trochotron from a co-ordinate system, which moves with the drift velocity, the electrons are acted upon by a homogeneous magnetic field only.

The phenomenon is probably related to a remarkable and unexplained phenomenen observed in magnetrons, *viz.* that a small current flows even above the magnetic field cut-off. More recently noise has been noticed in gaseous discharges in the presence of a magnetic field. The radio disturbances from the sun and the galaxy, known as solar noise and cosmic noise, may be of similar origin. In fact, the conditions in a trochotron and in the solar corona are rather similar. In the corona there is a magnetic field of the order of 20 gauss, the electrons having a velocity of about 100 volts (corresponding to a temperature of 1.000.000 degrees). Even the electronic density is of about the same order.

Hence, the noise and negative current observed in the trochotron, are probably very important phenomena, occurring as soon as electrons move in a magnetic field. A theory is given by MALMFORS.

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The Royal Institute of Technology.
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DESIGN AND PROPERTIES OF TROCHOTRONS

BY

T. WALLMARK

1. Introduction

A new electron tube, principally an electronic switch or selector, has been developed at the Royal Institute of Technology, Stockholm during the last four years.[1]) As the electrons in the tube, under the influence of an electric and a magnetic field, move in trochoidal paths, the tube has been given the name of *trochotron*. From the standpoint of electron behaviour inside the tube, it can be compared with a plane magnetron, working under non-oscillating conditions.

The tube to be described was suggested by H. ALFVÉN during his work with computation of the paths of charged particles in electric and magnetic fields, originally made for the explanation of certain cosmic phenomena (polar lights, magnetic storms etc.).

ALFVÉN found that under certain conditions existing in the space around the sun, beams of charged particles could be formed which radiated from the sun. The qualities of these beams suggested their utilization in some kind of switching or selecting device, where the movable part was the beam itself. Computations showed that the conditions necessary for the creation of the beam could be reproduced on a laboratory scale, and work was started to investigate the possible practical applications of such beams.

Some test tubes were made, measurements on which showed that the beams could be formed and shifted adequately, and that the voltages and currents were of a reasonable order of magnitude.

It is the purpose of this paper to present some of the work on the development of a number of tube designs utilizing these principles for different practical applications.

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[1]) H. ALFVÉN, H. ROMANUS: Nature, 160, pp 614—5, Nov. 1947.

2. Calculation of the electron path

A treatment of the theory for computation of the electron paths under the conditions found in the tube is given by H. ALFVÉN.[1])

In many practical cases, enough information about the behaviour of the electrons can be obtained in a very simple manner, by solving differential equations for the movement of the electrons under the somewhat simplified conditions given below. As such calculations have been frequently done in the past, especially in connection with magnetrons, only the essentials are given here.

In a constant, homogeneous magnetic field, perpendicular to a constant, homogeneous electric field, when the influence of space charge is neglected, the following equations for the motion of the electron are valid. See fig. 1.

$$m \frac{d^2x}{dt^2} = - ev_y B$$

$$m \frac{d^2y}{dt^2} = eE + ev_x B$$

$$m \frac{d^2z}{dt^2} = 0$$

where

E is the electric field strength

B is the magnetic field density

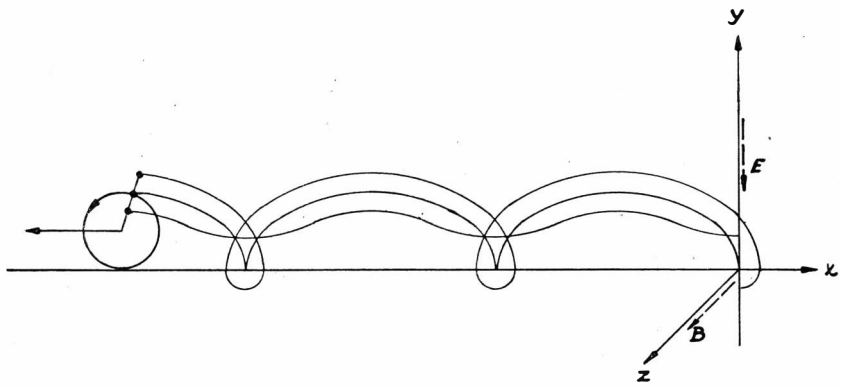


Fig. 1. Trochoidal paths.

[1]) H. ALFVÉN: On Trochoidal Electronic Beams and their Use in Electronic Tubes.

Integration gives

$$x = -\frac{E}{B} \cdot t + A \sin \frac{eB}{m} \cdot t$$

$$y = C - A \cos \frac{eB}{m} t$$

$$z = D \cdot t$$

This represents a trochoid in the $x - y$ plane, where A and C are constants depending upon the initial conditions. A motion in the z-direction, with constant velocity D, is superimposed.

$$A < \frac{mE}{eB^2} \text{ represents a prolate trochoid}$$

and

$$A > \frac{mE}{eB^2} \text{ represents a curtate trochoid}$$

As is indicated in Fig. 1, these curves are formed by a point on the radius of a rolling wheel.

If the electron starts from its origin without any initial velocity $A = \frac{mE}{eB^2}$ and the curve forms a cycloid. In this case the equations can be written

$$x = -\frac{E}{B} t + \frac{mE}{eB^2} \sin \frac{eB}{m} t$$

$$y = \frac{mE}{eB^2} - \frac{mE}{eB^2} \cos \frac{eB}{m} t$$

From these equations, the quantities typical for the electron path are directly obtained. Thus, the radius of the rotating motion of the electrons is

$$\varrho = \frac{mE}{eB^2}$$

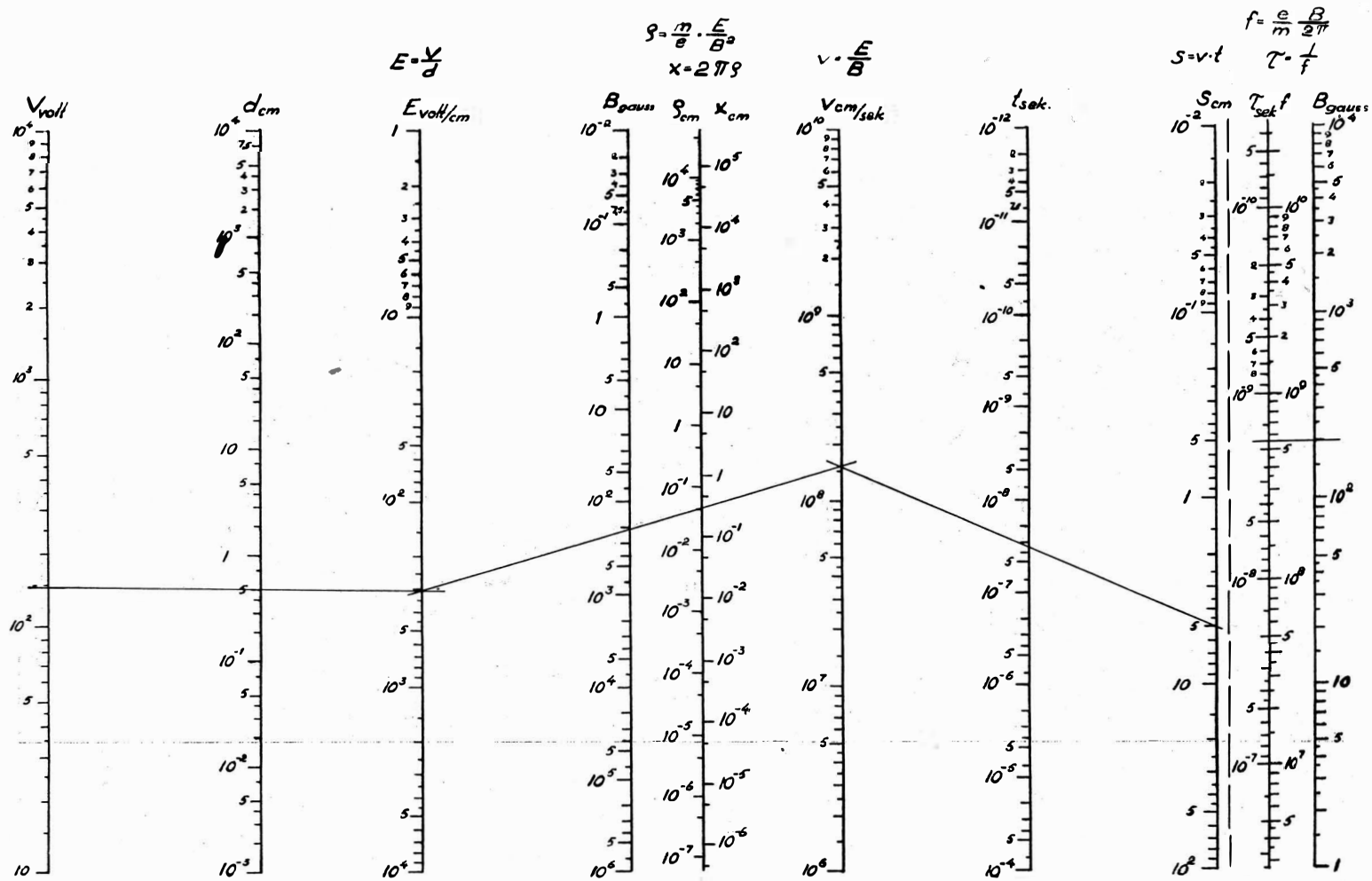


Fig. 2. Nomogram on cycloid paths.

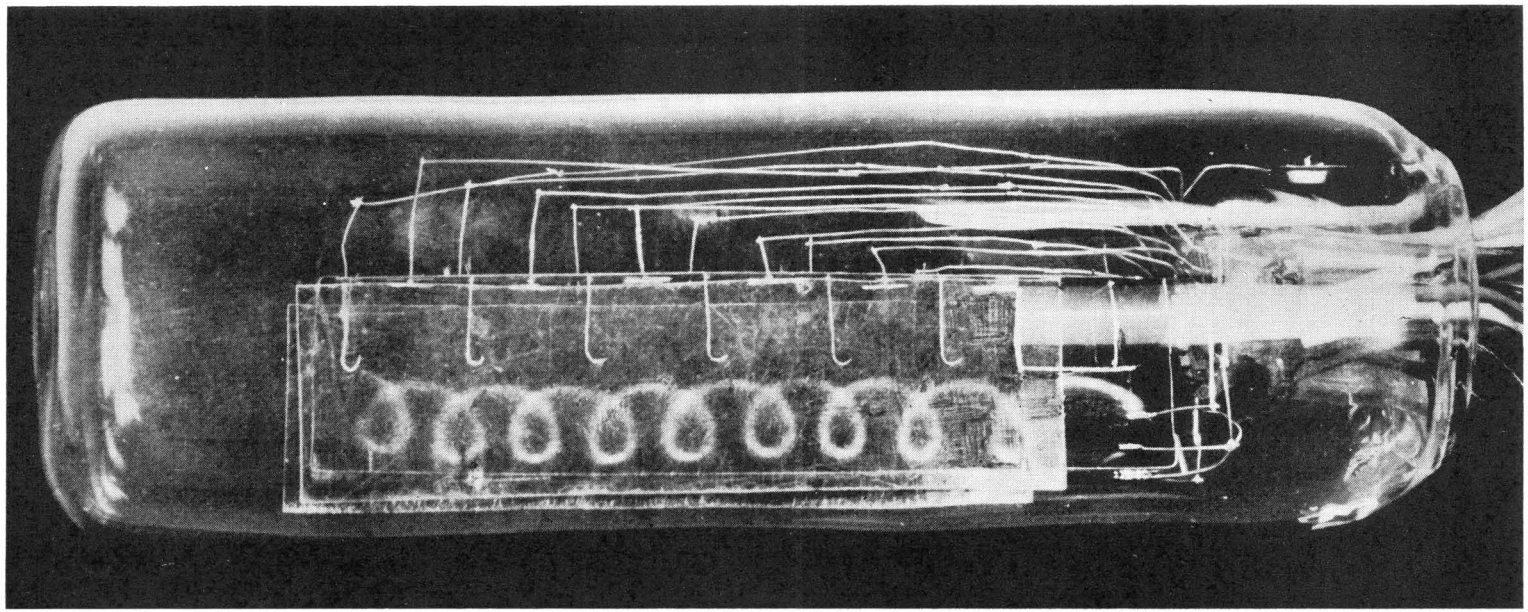


Fig. 3. The thread ray tube.

The average velocity through the tube

$$v = \frac{E}{B}$$

The rotation frequency

$$f = \frac{eB}{2\pi m} \text{ etc.}$$

The nomogram of Fig. 2 gives the most frequently used quantities as a function of voltage and magnetic field.

3. Visualization of the beam

As the tubes involved rather intricate electron optics and the complicated electric fields presented great difficulties for exact calculations, methods were sought for reaching a better understanding of the electron paths under different conditions.

One of the most valuable of these, visualization of the electron path by thread rays, will be described in more detail.

With this method, first given by WEHNELT,[1]) and further developed by BRÜCHE,[2]) some gas is let into the tube (appr. 10^{-3} mm Hg). This causes the electrons to excite and ionize the gas molecules along their path, which becomes visible as a narrow blue luminous »thread ray».

Fig. 3 shows an experimental tube in which the paths of Fig. 5 were photographed. To make the photographs of Fig. 5 clearer, the electrodes are filled in with white. The tube structure is similar to the trochotron described later.

The spade-like electrodes *s* 1, *s* 2, *s* 3, *s* 4, *s* 5, and *s* 6 in fig. 4 are

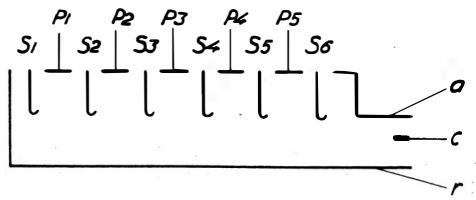


Fig. 4. The thread ray tube.

[1]) A. WEHNELT: Z. Phys. Chem. Unterr. 18, 193, 1905.

[2]) E. BRÜCHE: Z. Phys. 64, 168, 1930.

called spades, and have the ability, as will later be shown, to »dig up» the beam current, which is collected by the plates p 1, p 2, p 3, p 4 or p 5. The anode a and the cathode c form a simple electron gun, which sends the electrons in trochoidal paths, rolling along the rail r. The magnetic field is perpendicular to the paper and the electric field is maintained between the spades and the rail. The photographs are taken with a magnetic field of 50 gauss and a voltage of 150 volts on the spades, plates and anode. The rail is at cathode potential.

In Fig. 5 the electrons follow a trochoid, as can be expected from theory. The boundary conditions at the cathode define the trochoid to become curtate. The trochoid follows an equipotential surface through the tube. Where the surface passes out through the narrow space between two electrodes, the electrons are collected in proportions determined by the field structure.

Suppose the voltage of one of the spades. e. g. s 3, is lowered from 150 volts to zero. In this case the equipotential surface is shifted over to pass between s 3 and p 3. Consequently, the beam also shifts, enters the box between s 3 and s 4, and is ultimately collected by p 3. The currents to different electrodes as a function of spade voltage are shown in Fig. 6. When the beam leaves spade 1, spade 3 starts to draw current, which, after having reached a maximum at a voltage corresponding to the trochoid potential, decreases and shifts over to p 3.

The described process can be followed in Fig. 5 a—f.

Fig. 5 f—l show a case which is important in pulse-stepping. First, the voltage on plate p 3 is lowered, causing the electrons to go to spade s 4. The potential of spade 4 is hereby lowered, causing the electrons to pass around and enter the next box, where they are collected by p 4.

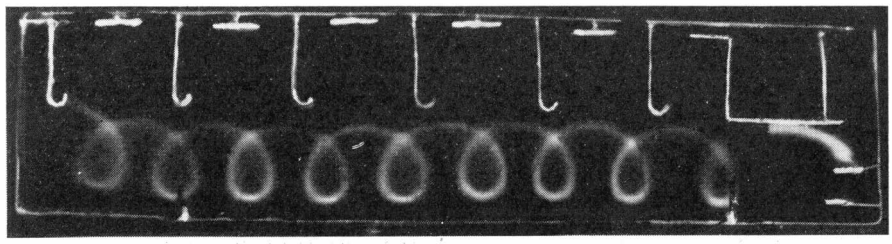
The current diagram, when the voltage of a plate is lowered, is shown in Fig. 7.

In the photographs shown, the radius of the electron path has been kept rather large for the sake of clarity, i. e. the magnetic field rather weak. For practical reasons it has been found advantageous to diminish the radius, i. e. to run the tube with higher magnetic field. In this case the path is more similar to Fig. 8.

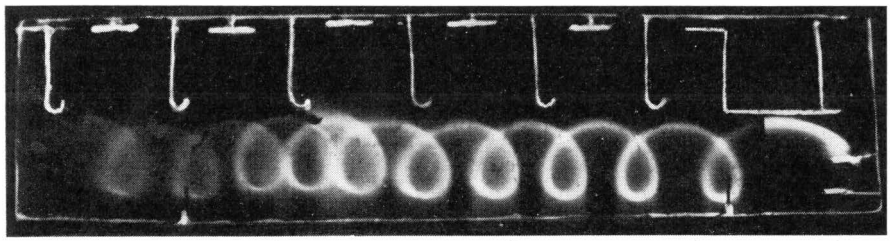
In the actual tube, the beam is still more homogeneous, since the cathode emits over a large area, whereas the photographs show emission from a point source. Space charge also tends to smoth the beam structure in the actual tube.

The photographs show, however, a very close resemblance to the calculated paths, and give a good picture of what is happening in the tube.

The description of the tube types developed will be based on the straight ten-box trochotron which has the widest application and shows most of the fundamental qualities. A detailed description



a



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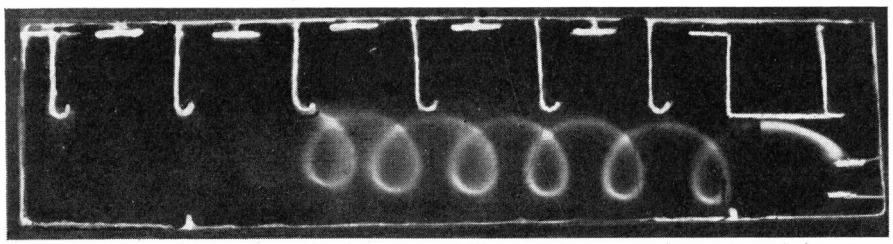

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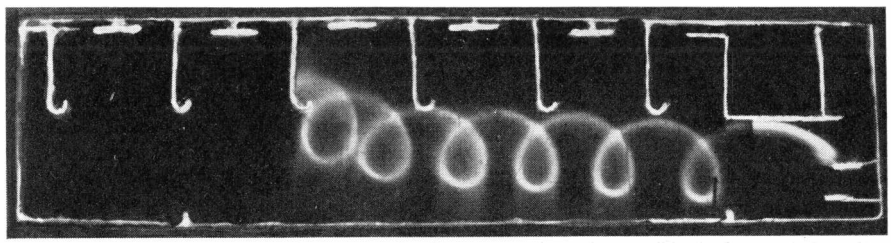

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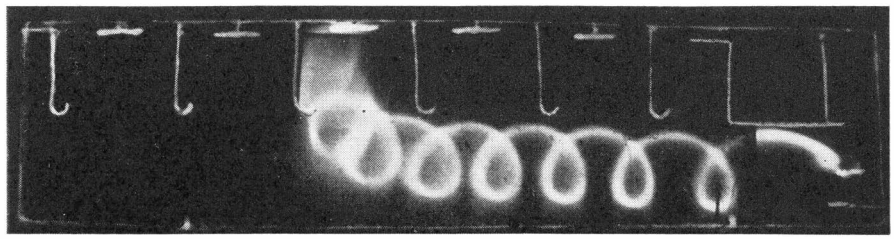

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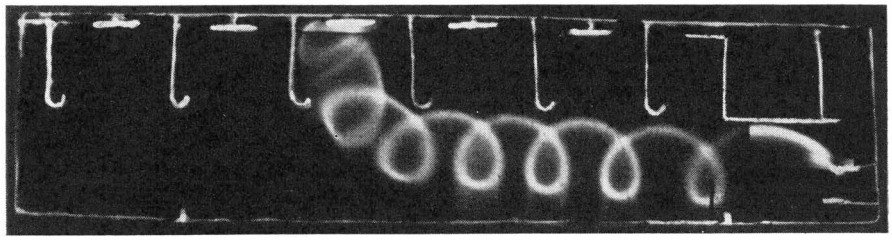

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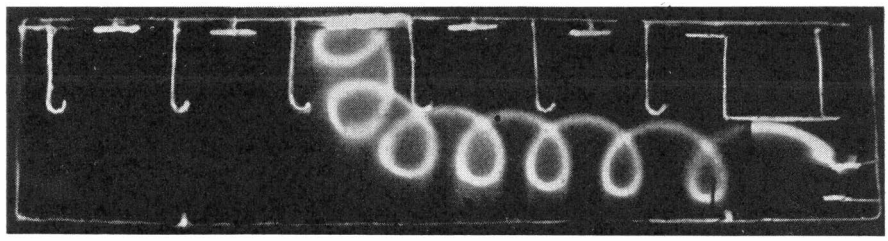

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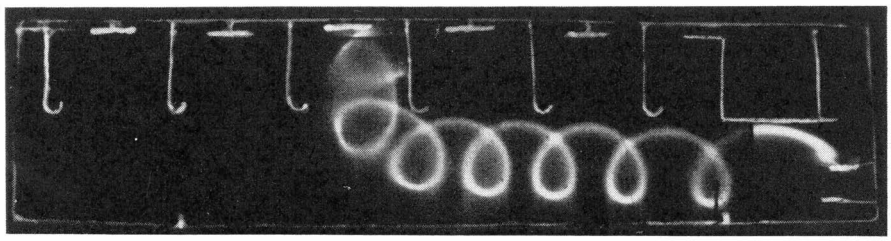
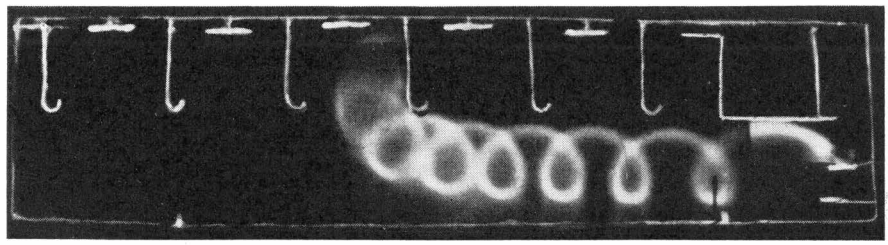


i



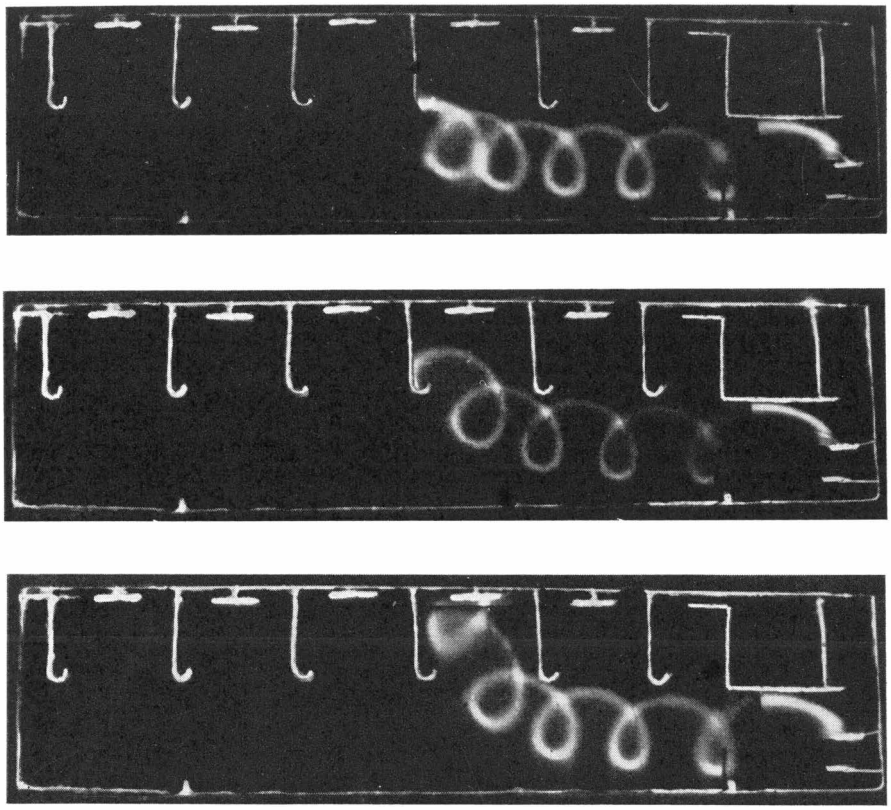


Fig. 5. Photos of electron paths.

Photo	V_{s3} volts	V_{p3} volts	V_{s4} volts
a	150	150	150
b	110	150	150
c	90	150	150
d	70	150	150
e	30	150	150
f	0	150	150
g	0	70	150
h	0	0	150
i	0	0	110
j	0	0	80
k	0	0	40
l	0	0	0

All other spades and plates at 150 volts.

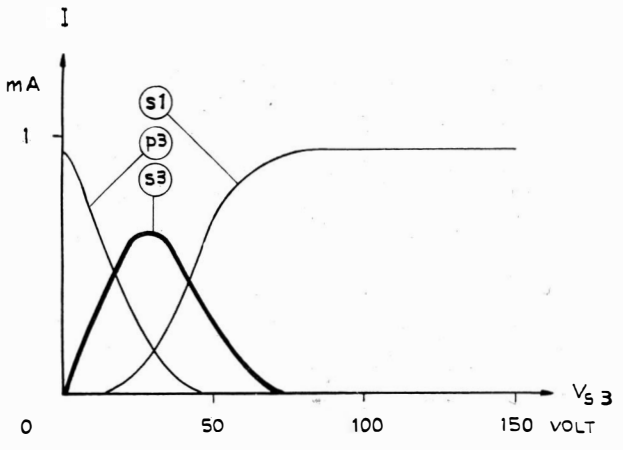


Fig. 6. Current diagram when the spade voltage is varied.

$V_{s1} = V_{s2} = V_{s4} = V_{s5} = V_{s6} = 150$ volts

$V_{p3} = 150$ volts.

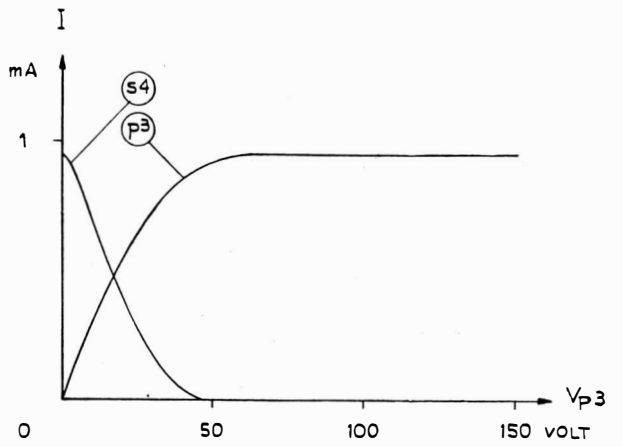


Fig. 7. Current diagram when the plate voltage is varied.

$V_{s1} = V_{s2} = V_{s4} = V_{s5} = V_{s6} = 150$ volts

$V_{s3} = 0$ volts.

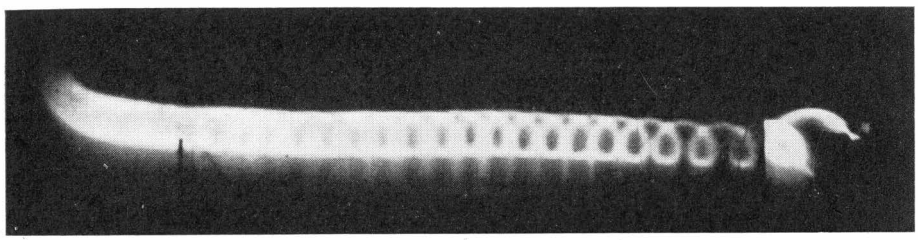


Fig. 8. Electron path with higher magnetic field,

of the performance of the tube is found in the papers by H. ALFVÉN and L. LINDBERG.[1])

Originally intended for counting purposes in connection with GEIGER-MÜLLER tubes, the trochotron was developed for the pulse-stepping method described by L. LINDBERG and briefly mentioned above.

To ascertain the most suitable form for the tube and its elements, a series of some 50 tubes has been built, on which successive improvements have been introduced. The results of these experiments may be summed up as follows.

4. Operating conditions

A trochotron suited as pulse counter is shown in Fig. 9. Normal operating conditions are

electrode voltages	200 volts
magnetic field	200 gauss
beam current	1 mA

These figures, however, are largely arbitrary and may be varied considerably, up or down, while still giving satisfactory operation, provided that certain elementary conditions are fulfilled. (The height of the trochoid should not exceed the interelectrode space etc.) With the cited figures, it is possible to keep down stray currents to less than 0.1 %, provided that the precautions discussed in 6, are taken. Only to the spade next to the box which receives the beam, are higher currents unavoidable.

From the point of view of high speed pulse stepping it is desirable to increase the current and decrease the interelectrode capacitances.

[1]) H. ALFVÉN: On Trochoidal Electronic Beams and their Use in Electronic Tubes.
L. LINDBERG: Design of Trochotron Circuits.

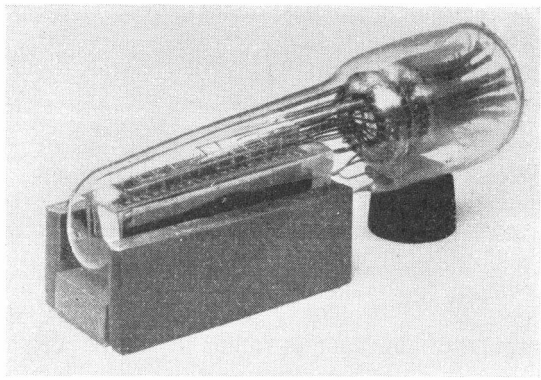


Fig. 9. Ten-box trochotron with permanent magnet.

The transit time from the cathode to the furthest end of a tube of approximatively 5 cm length is below 10^{-7} sec, whereas the time-constant of spade circuits is of the order of microsec. Thus the limit of high speed operation is set not by the electron transit time, but by the capacitances between electrodes, and from electrodes to ground, and by the maximum obtainable current through the tube.

The capacitances are largely set by practical considerations, since, with the present construction, the main part is due to the rather long support wires. Here, an improved construction should reduce the capacitances appreciably.

It has been found that shielding and balancing, by suitable arrangement of the support wires, may bring down the influence of the capacitances.

The maximum obtainable beam current is set by space charge considerations, as described by E. ÅSTRÖM.[1]) It has been found that when the space charge reaches a critical value, instabilities in the beam appear which jeopardize stable operation of the tube.

5. The size of the electrode system

During the experiments, an interelectrode distance of 5 mm has been used. This means that the entire channel along which the trochoid proceeds is 5 mm wide. This is, however, entirely arbitrary, as only the relative proportions between trochoid and electrode size are of importance. It has also been found that the performance of

[1]) E. ÅSTRÖM, Experimental Investigation on an Electron Gas in a Magnetic Field.

the tube can be well reproduced on another scale (reduced or enlarged) provided that the electron path, by suitable choice of voltage and magnetic field, is changed on the same scale.

With everything else kept constant, the maximum current varies linearly with the dimensions parallel to the magnetic field. The interelectrode capacitances, however, vary in the same proportion, only the reduced influence of end effects being gained by making the tube wider. This meaning, at the same time, a magnet with wider gap, the gain is questionable. Reducing or enlarging the size of the tube by simple proportioning should not affect the maximum current, according to present theory.

6. The dimensioning of the boxes

As the cathode is not a point electron source, but emits over a large surface and in different directions, the beam will not be sharply defined. The diffuse boundaries of the beam have a tendency to cause »leak currents» to electrodes which should not take current. The positive spade in a box where the beam terminates is especially liable to this effect, and the current versus voltage diagram takes the form of Fig. 10.

This is extremely undesirable, as it may switch down the positive spade in a box before a pulse has reached the plate, as a consequence of the voltage drop in the spade resistor caused by the leak current.

These leak currents are, of course, particularly large if the box is narrower than the extension of the trochoidal beam itself. Increasing the distance between the spades, however, does not eliminate the leak current.

The lower edge of the spades should be given a large curvature, in order to avoid strong irregular fields. If this is not done the electric field around the edge is far from constant, and strong leak currents appear.

Another source of leak current is collisions between electrons and residual gas molecules in the tube. If an electron, by an impact, loses some of its energy, its path is transferred to higher equipotential surfaces, and thus may be collected by an electrode at a higher voltage than the rest of the beam.

Collisions between electrons should also change the paths in a similar manner. Calculations show, however, that the probability of such collisions is extremely small.

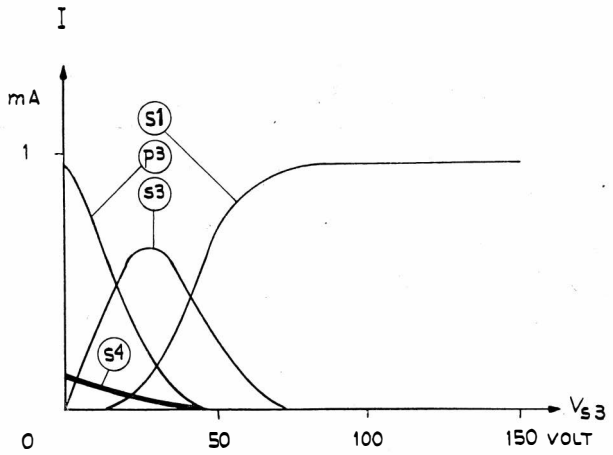


Fig. 10. The leak current.
$V_{s1} = V_{s2} = V_{s4} = V_{s5} = V_{s6} = 150$ volts
$V_{p3} = 150$ volts

To avoid field distortion in the furthest box, the rail should be lengthened somewhat behind the first spade. This gives similar electric field conditions in all boxes.

It is essential that the depth of the boxes should exceed the distance between the spades, so that the beam is not influenced by the potential variations of a plate in a box which the beam does not enter.

The ability of a spade to pass the current completely from one box to another, when its voltage is lowered to near zero volts, is of great importance in the practical application of the tube. The smooth curvature of the lower edge of the spades aids in this respect, as does also the introduction of side flaps later described.

7. Sides

In order to keep the beam together laterally (in the direction of the magnetic field) in some way, a force must be introduced to keep the electrons in a stable path in the centre of the tube. This effect is produced by the sides of the rail which give electric equipotential surfaces, as shown in the cross-section of Fig. 11. Near the sides the electric field has two components, one vertical, which gives the trochoidal motion, and one horisontal, which returns the electrons to the centre of the cross section. For the same reason the spades are given side flaps as shown in Fig. 12, which, when the beam enters

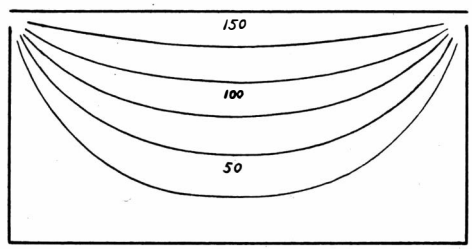


Fig. 11. Cross-section of the trochotron, equipotential surfaces.

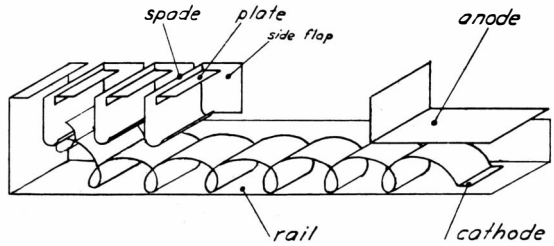


Fig. 12. Perspective sketch of the electrode system.

a box, constitute a direct continuation of the rail. As mentioned, they also increase the ability of the spade to affect the switch-over of the beam, i. e. to influence the electric field between the spade and the rail.

8. Insulation

If the interstices between the electrodes are left open, some parts of the electrons leak out and are collected by support wires outside the electrode system, causing irregular leak currents. In order to avoid this, all interstices are covered by mica, and care is taken to keep the space between mica and metal small enough on all critical points.

Such a point has been found to be the upper edge of the spade, and in order to further reduce the probability of leak currents here, part of the plate is bent down, thus preventing the main part of the electrons from reaching the interstice.

As the potential of the mica is not stable, it is desirable to keep the parts of the mica exposed to the electron beam as small as possible. If this is not done, instablilities in beam current and current distribution may appear.

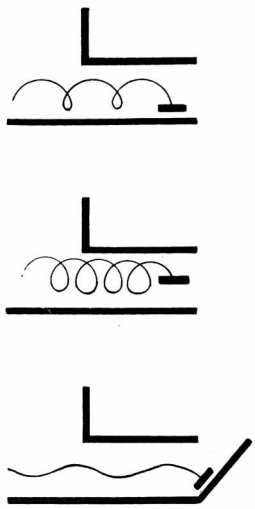


Fig. 13. The anode-cathode arrangement.

9. The electron source

The anode has been given the form indicated in Fig. 13, and is made so long that the field from the other electrodes does not reach the cathode and there influence the emission. The function of the anode is to control the emission, but only on exception to collect electrons.

The form and location of the cathode is the main factor determining the structure of the beam. A cathode located near the rail gives nearly cycloidal electron paths, whereas the paths from an elevated cathode, on account of the field concentration, are more like a curtate trochoid. A prolate trochoid is obtained if the field near the cathode has a strong component in the direction of the beam. This is the case when the rear end of the rail is bent up towards the anode, in the vicinity of the cathode.

A curtate trochoid is the most desired path form, other paths having a tendency to distort the spade characteristics as will be described later.

This bending up, however, cannot be done too near the cathode, although it is desirable to shield the cathode from the influence of stray fields from the glass or support rods.

In many cases it is necessary to keep the emission constant within certain limits (e. g. $\pm$ 25 %) in spite of variations in line voltage, cathode temperature etc., in order that the tube shall function satisfactorily. This is achieved by a form of grid control. The grid action is performed by a grounded trough, in which the cathode is mounted. The cathode is connected to the ground via a bias resistor,

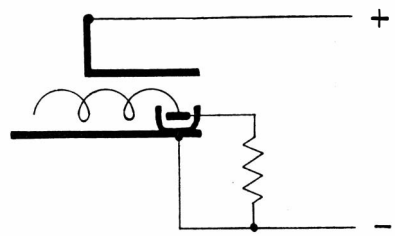


Fig. 14. The cathode trough for stabilizing the beam current.

and the voltage drop in this resistor, due to the emission current, stabilizes the emission quite effectively.

The insertion of this trough around the cathode also has the result that the emission from the edges and rear side of the cathode is prohibited, which means a better defined beam and elimination of a great deal of the leak current.

The magnetic field of the heater current, and the heater voltage of directly heated cathodes, gives the electrons a deflection towards the positive end of the cathode. Thus, the electrons have a winding path through the tube. In some cases this is not desirable. An indirectly heated cathode with little magnetic action is to be preferred.

10. Deformation of the spade characteristics

For the described application of the trochotron it is desirable that the spade characteristic has a rather high peak and does not show any irregularities. As is shown by experiments, the electron path should then be a curtate trochoid. In the case of a cycloid or a prolate trochoid, disturbances of the spade characteristic appear, due to the following electron-optical considerations.

When the beam is in the stable position in a box, and the voltage of the spade close by is lowered in order to pass the beam to the next box, it may happen that the field below the spade takes such a form for certain electron paths that the electrons hit the spade only within a very limited voltage range. Especially in the case of a prolate trochoid the electron ray may be reflected in the vicinity of the spade, and to a large extent, miss it. This causes the current to the spade to decrease below normal, and the spade characteristic becomes distorted. Simultaneously, corresponding distortions appear in the current curves of the adjacent plates. As an experimental investigation has shown, the more prolate the path, the more pronounced is this effect.

The curves may be distorted in a similar manner by secondary emission from the spade. This may, however, be reduced by sufficient cleaning of the surfaces, and the choice of a suitable material.

In order to indicate the box in which the beam terminates it is possible to connect small glow lamps in series with the plates. As this is sometimes inconvenient with regard to small dimensions and low cost, it has been found practical to paint the plates with flouores-

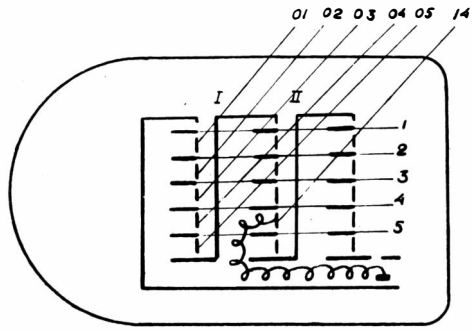


Fig. 15. Coordinate tube.

cent powder. When looking at them through small perforations in the rail it is possible to see how the beam moves. Due to the dark interior of the tube, this is practicable even in daylight, and at comparatively low voltages.

11. The coordinate tube

The special features of the trochoidal beams make possible the design of a tube where the beam can be directed along two coordinates. Fig. 15 shows the principles of construction. The tube contains a number of rows, in this case three, each consisting of five boxes. The beam can be switched to any of the plates, e. g. 14, by giving zero potential to the appropriate spades, in this case I and 4. In a manner similar to the usual trochotron the beam can be switched by pulses if desired, although here it is necessary to lock the beam by supplying parts of the beam current to both one of the vertical and one of the horizontal spades. Only practical considerations as to the number of leads through the glass envelope have prevented the building of still larger tubes.

12. By-spade

Another form of the tube which offers new application possibilities uses a second spade in each box, called a by-spade. The design in Fig. 16 describes the principle. The by-spades are long enough to dominate the electric field inside a box, but short enough to have a negligible influence on the beam outside the box, especially as

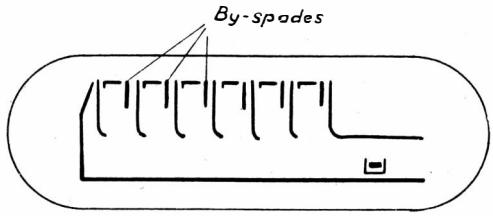


Fig. 16. By-spade arrangement.

they are not equipped with lower end radii or side-flaps. Thus a box can be closed to the beam independent of the voltage of the plate, with the by-spade collecting all electrons when given a suitable low voltage. In the same manner the box can be opened by giving the by-spade a high enough voltage, e. g. the same as the near spade. In this case, all electrons reach the plate as though the by-spade did not exist.

By giving the by-spades suitable form it is possible to change their characteristics with regard to cut-off, current collection a. s. o.

One of the applications of this form of the tube is described by LINDBERG.

13. Construction of the tubes

During the development work, all tubes were made on a laboratory scale, entirely manually, and without any attempt to adapt the design to industrial methods. In the adaption of the tube for large scale production, changes of the exterior may be advantageous.

Magnetic material has been avoided in the manufacture of the electrodes, which have been made of molybdenum. The support wires outside the electrode system have, however, for practical reasons, been made of nickel without any noticeable difficulty. The problem of getting the requisite number of leads through the glass for the many electrodes has hitherto been solved by ring seal stems. As this makes the leads inconveniently long, from a high frequency point of view, other constructions should be sought.

Tolerances, a factor of paramount importance in all electron tube manufacturing, seem to be considerably less critical than in ordinary tubes.

The magnetic field may most conveniently be supplied by a small

permanent magnet. The tube makes no rigorous demands on homogeneity and uniformity of the magnetic field which allows the use of a very small magnet.

The author wishes to acknowledge his debts to Prof. H. ALFVÉN for his inspiring leadership of our research group, and to all collaborators whose cooperation has made possible the successful completion of the work.

Electronics Laboratory.
The Royal Institute of Technology.
Stockholm.

DESIGN OF TROCHOTRON CIRCUITS

A SUMMARY BY

L. LINDBERG

Principal circuits

1. *The basic plate-pulsed circuit*

ALFVÉN, in his paper,[1]) describes how the trochoidal beam can be locked to a box by high spade-resistances, and stepped by negative voltage pulses applied to the plates, in a circuit similar to Fig. 1. This principle is further developed here. The circuit cited needs very well-defined pulses, with a duration just equal to the time for a switch-over, for if a pulse remains after having displaced the beam one step, the plate in the new box will also be at a low voltage, enabling the beam to continue one or more steps until the pulse vanishes. The circuit is also critical to the other operating data, since these influence the time required for one switch-over. Consequently, it is critical and unreliable to operate. To avoid these difficulties, the

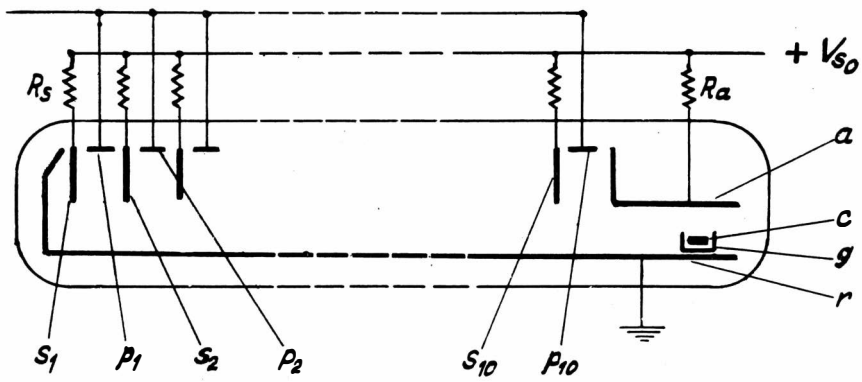


Fig. 1. The basic, plate-pulsed, stepping circuit.

[1]) See H. ALFVÉN, On Trochoidal Electronic Beams and their Use in Electronic Tubes (Trochotrons.)

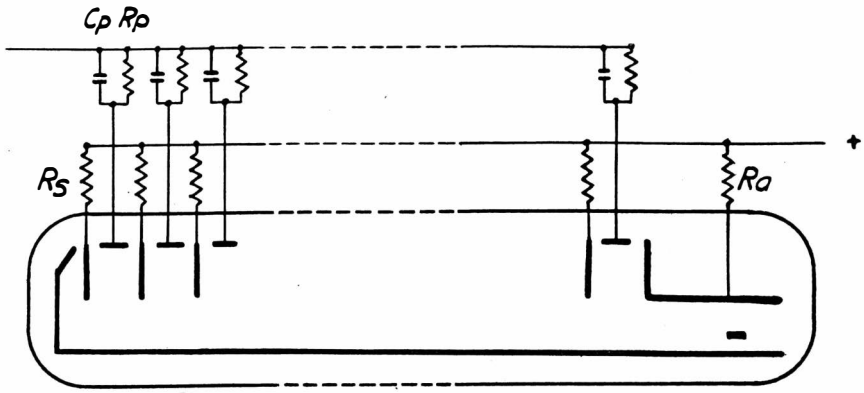


Fig. 2. R-C-coupled, plate-pulsed, stepping circuit.

voltage of the next plate, at every switching position, must be prevented from passing below a critical value, the »stepping voltage». This is achieved by the following two methods.

Fig. 3

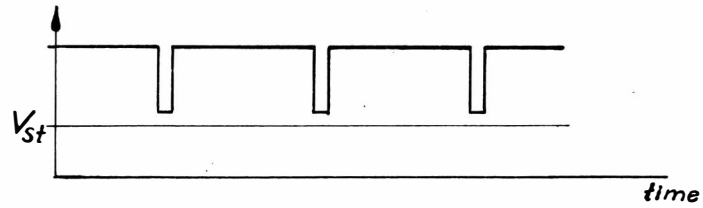


Fig. 4

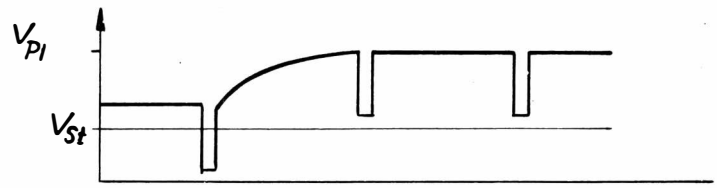
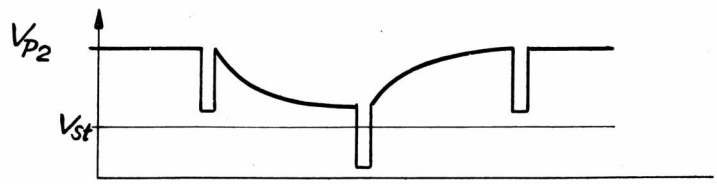


Fig. 5



Locking position, box: 1 2 3 4

Fig. 3. Plate supply voltage, pulsed. Fig. 4. Voltage of plate 1. Fig. 5. Voltage of plate 2.

2. *R-C-coupled plate-pulsed circuit*

The method, first employed by HAMBRAEUS, by which to make a stable counter, was to insert a R-C-circuit in series with each plate, as shown in Fig. 2, and to feed them all with negative exciting pulses, Fig. 3. Supposing the beam first locked to box 1, the plate current will cause a voltage drop in its plate-resistance, which, however, is insufficient to shift the beam to box 2. When a pulse arrives, it brings down the plate voltage below the stepping value, *Vst*, (See Fig. 4). The voltage of spade 2 suddenly drops, and shifts the beam to box 2. Here, due to the plate capacitance, the plate voltage cannot immediately drop below the stepping value, even if the pulse remains for a moment, as shown by Fig. 5. In this way, a broad margin for the pulse duration, as well as for the operating data may be achieved.

3. *Push-pull coupled plates*

By this method, first suggested by BJÖRKMAN, the plates are connected into two groups, odds and evens, and fed in push-pull from the anodes of a common »scale of two», which, in its turn, is operated by the pulses, according to Fig. 6. Then the beam can remain stable in a box where the plate is positive, but when the trigger shifts the plate voltages, it will step to the next box, and, as that plate simultaneously has become positive, it will remain there. Thus a very reliable circuit is acquired.

The plate voltages do not necessarily need to rise and fall very steeply. A sine-wave for instance, may also step the beam. The

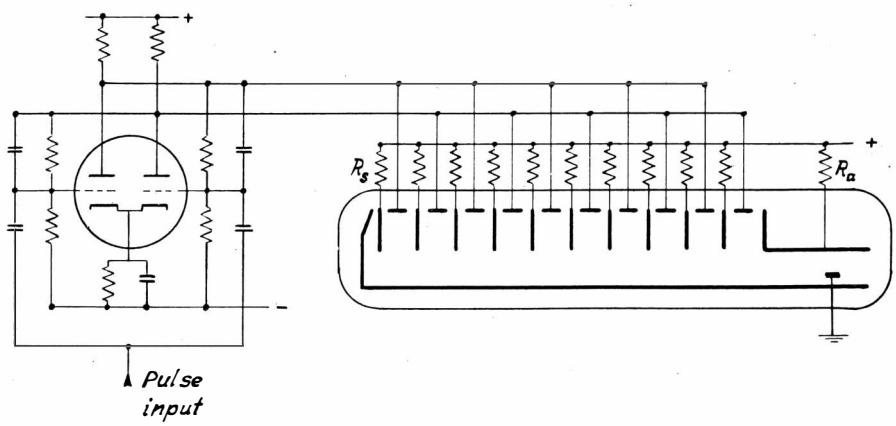


Fig. 6. Push-pull-coupled, stepping circuit.

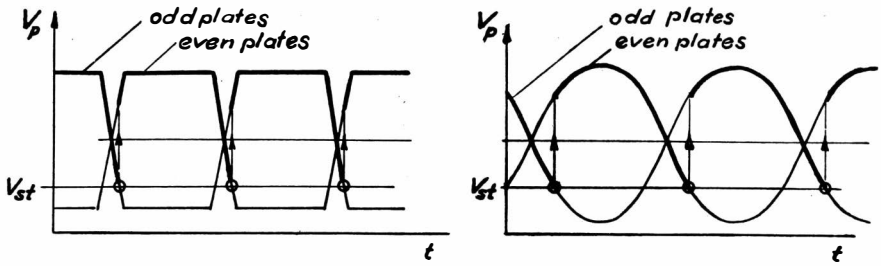


Fig. 7. Plate voltages.

only necessary condition is, that the two groups of plates should never be below the stepping voltage simultaneously, as indicated by Fig. 7. The points, where stepping occurs, are marked with circles.

Pulse technique

4. *General remarks*

After this survey of the general principles, we are now going to discuss the processes quantitatively with the aid of suitable tube characteristics, in order to find out the most suitable operating conditions.

In the following, we will denote all voltages with respect to the rail as zero, partly because these voltages determine the field structure within the tube and thus give a clear understanding of the tube functioning, but also because the cathode voltage, in some applications, is varied, in order to shift the beam position. In the pulse-stepping circuits, a cathode resistance is usually employed to stabilize the emission, so that the cathode is self-biased, and takes a positive voltage.

The electron beam in a trochotron degenerates while travelling through the tube. As is pointed out in the papers by ÅSTRÖM and WALLMARK,[1]) the degeneration manifests itself in three ways: noise, widening of the beam and currents to electrodes negative with respect to the cathode. These effects all seem to vary with the operating conditions in the same way.

In practical use for pulse-stepping it is essentially the widening of the beam that becomes disturbing, since it causes leak currents to

[1]) T. WALLMARK. Design and Properties of Trochotrons. E. ÅSTRÖM: Experimental Investigation on an Electron Gas in a Magnetic Field.

positive spades, and broadens the characteristics of the spades farthest from the cathode. The rail current becomes important only if it considerably reduces the beam current, and the noise, as such, becomes very important in applications to telephony.

In order to hold the disturbing currents in relation to the total beam current within a safe value, when choosing the operating conditions for a given trochotron, we have to keep the exponent

$$\frac{V_a^{3.75}}{I_e^{1.5} \cdot B^3 \cdot L}$$

above a certain value. (cf. E. ÅSTRÖM, *loc. cit.*).

Here,

V_a = voltage of anode and spades.
I_e = total beam current.
B = magnetic field strength.
L = length of the beam path.

Further, the height of the beam, which is proportional to $\frac{V_a}{B^2}$ must be safely within the tube space. Thus, if V_a is already chosen, B must be chosen as a compromise between these two claims, and if we change V_a, we should change B proportional to $V_a^{1/2}$. Then the allowable current varies proportional to $V_a^{3/2}$, (if we do not change B, it varies as $V_a^{5/2}$). As becomes evident later, the ratio $\frac{I_e}{V_a}$, the »conductivity» of the trochotron, is a figure of merit. As this is proportional to $V_a^{1/2}$, rather high supply voltages are favourable, especially for fast operation.

Numerical examples of voltages, currents and times are to be found in the chapter on applications and are only intended to show their order of magnitude in the trochotrons hitherto used.

Study of the Stepping Process

5. *The locking of the beam*

We suppose the beam locked to an arbitrary spade, s_N in Fig. 8. Fig. 9 shows the current-versus-voltage-diagram of that spade, when all other spades and plate p_N are at the positive supply voltage. (The voltages of the other plates are of no importance.) The spade resistance line intersects the spade characteristic at three points,

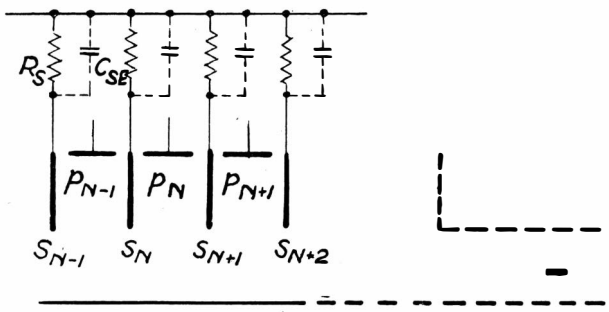


Fig. 8.

A, B, C, of which A and C are stable. In the locking position at C, the spade has the »locking voltage» V_L and takes the »locking current» I_L. The rest of the beam, $I_B - I_L$, goes to plate N. From this characteristic a lower limit for the spade resistance is established, corresponding to the line, tangential at D. Fig. 10 shows the characteristic of spade $N+1$ when spade N is at V_L and plate N positive. (If spade N were fully positive, it should be a copy of Fig. 9, but now it is broader.)

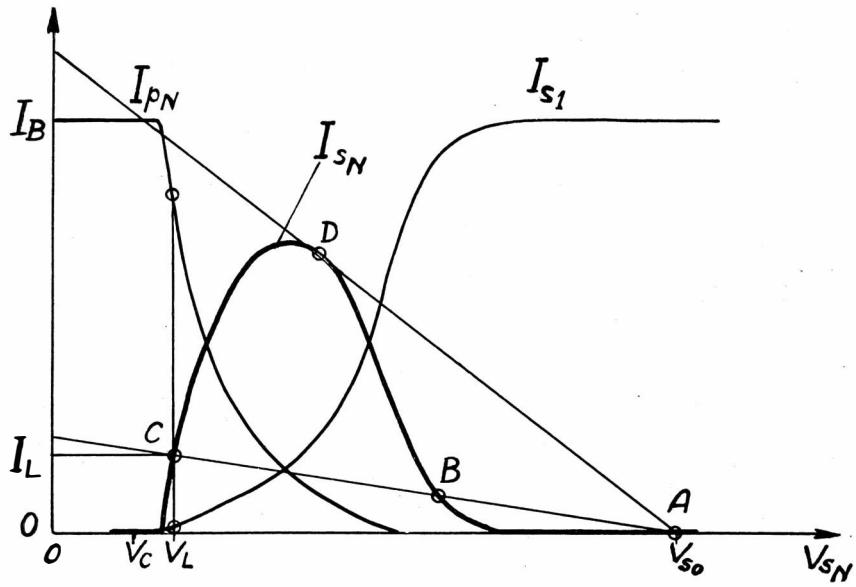


Fig. 9. Current distribution when the voltage of a spade is varied, while all other spades and plates are at positive supply voltage.

$V_{s\,N-1} = V_{s\,N+1} = \ldots . V_{s\,10} = V_a = 200$ volts $V_{p\,N} = 200$ volts.

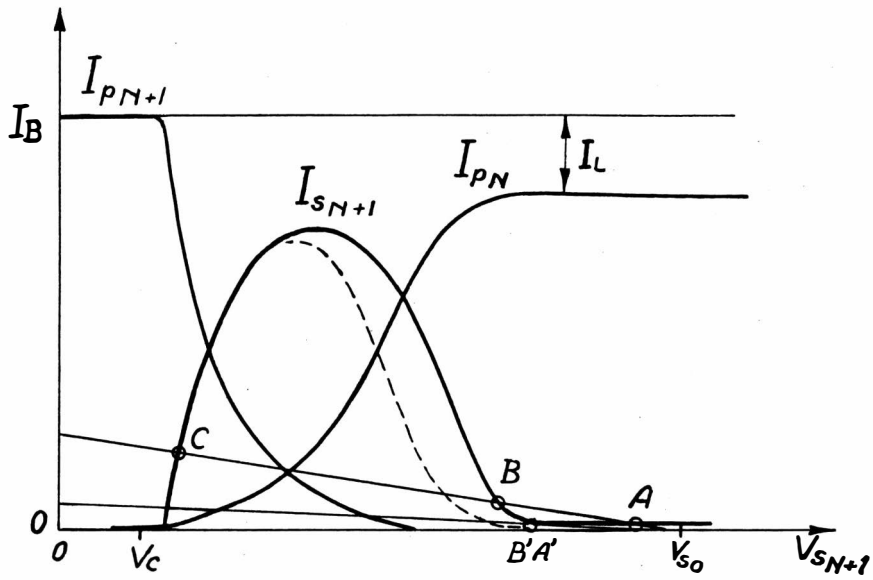


Fig. 10. Current distribution when the voltage of a spade is varied, while the previous spade is lowered to the locking voltage.

$$V_{s\,N} = V_L$$

$$V_{s\,N+2} = \ldots . V_{s\,10} = V_a = 200 \text{ volts}$$

$$V_{p\,N} = V_{p\,N+1} = 200 \text{ volts.}$$

We wish spade $N+1$ to have the voltage of point A in Fig. 10. Due to leak current, as described by WALLMARK, the characteristic in Fig. 10 does not quite fall to zero at positive voltages. If we increase the spade resistances, the points A and B will approach each other and will finally coincide at A' B'. Then the upper limit to the spade resistance is obtained.

In the following, we assume rather large spade resistances, about 2—10 times the lower limit value.

6. *Switch over*

We shall now discuss the shifting of the beam. For the sake of simplicity we shall first discuss the push-pull-coupled circuit and assume the plate voltages to have ideal rectangular wave forms. The capacitances of a spade to other electrodes will be represented only as a capacitance to earth, C_{SE} (a few $\mu\mu F$) shown dotted in Fig. 8.

When we lower the voltage of plate N, the right part of the

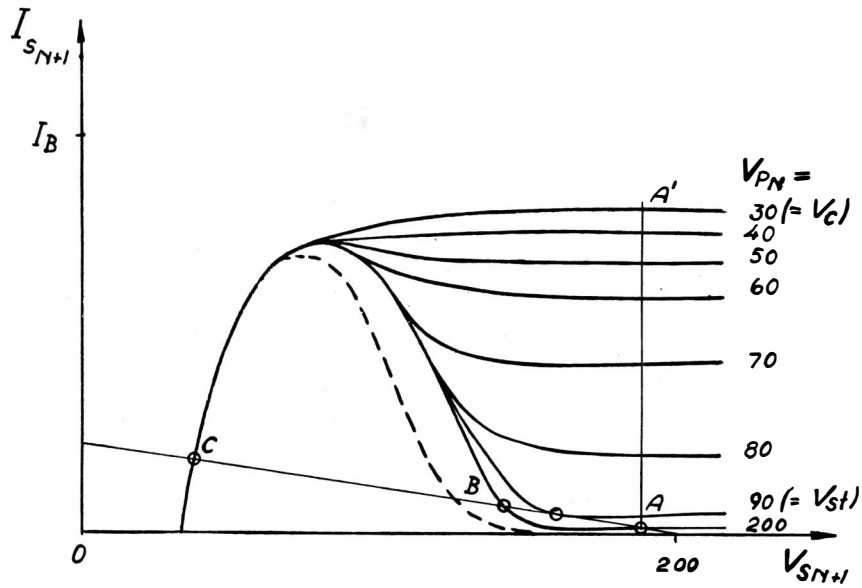


Fig. 11. Characteristic of a spade for various voltages of the previous plate, when the previous spade is at locking voltage

$$V_{s\,N+2} = \ldots . V_{s\,10} = V_a = 200 \text{ volts.}$$

characteristic of spade $N+1$ in Fig. 10 is raised, as shown in Fig. 11. The stepping voltage is just that, for which A and B coincide and vanish, leaving only one stable point at C.

At first we assume the plate voltage to be brought down to, or brought slightly below, the cathode voltage, as in Fig. 13 a. The operating point is displaced from A to A' at the highest characteristic of Fig. 11, as in Fig. 12 a, that is, the whole current to plate N hits spade $N+1$ and begins to charge its capacitance to earth, causing the spade voltage to fall rapidly until the stable equilibrium at C is reached. Fig. 14 a shows the current to spade $N+1$ and Fig. 15 a its voltage as functions of time. Corresponding to Fig. 12 a the current, at first, is large and constant, equal to $I_B - I_L$ causing the voltage to fall almost linearly, initially with the slope $\frac{I_B - I_L}{C_{SE}}$. The current to spade N does not cease before spade $N+1$ has reached a fairly low voltage. Then the voltage of spade N rises exponentially to the supply voltage, as shown also in Fig. 15 a. When the input plate-voltages are shifted the next time, the beam strikes spade $N+2$ and its voltage will drop in the same manner as described above.

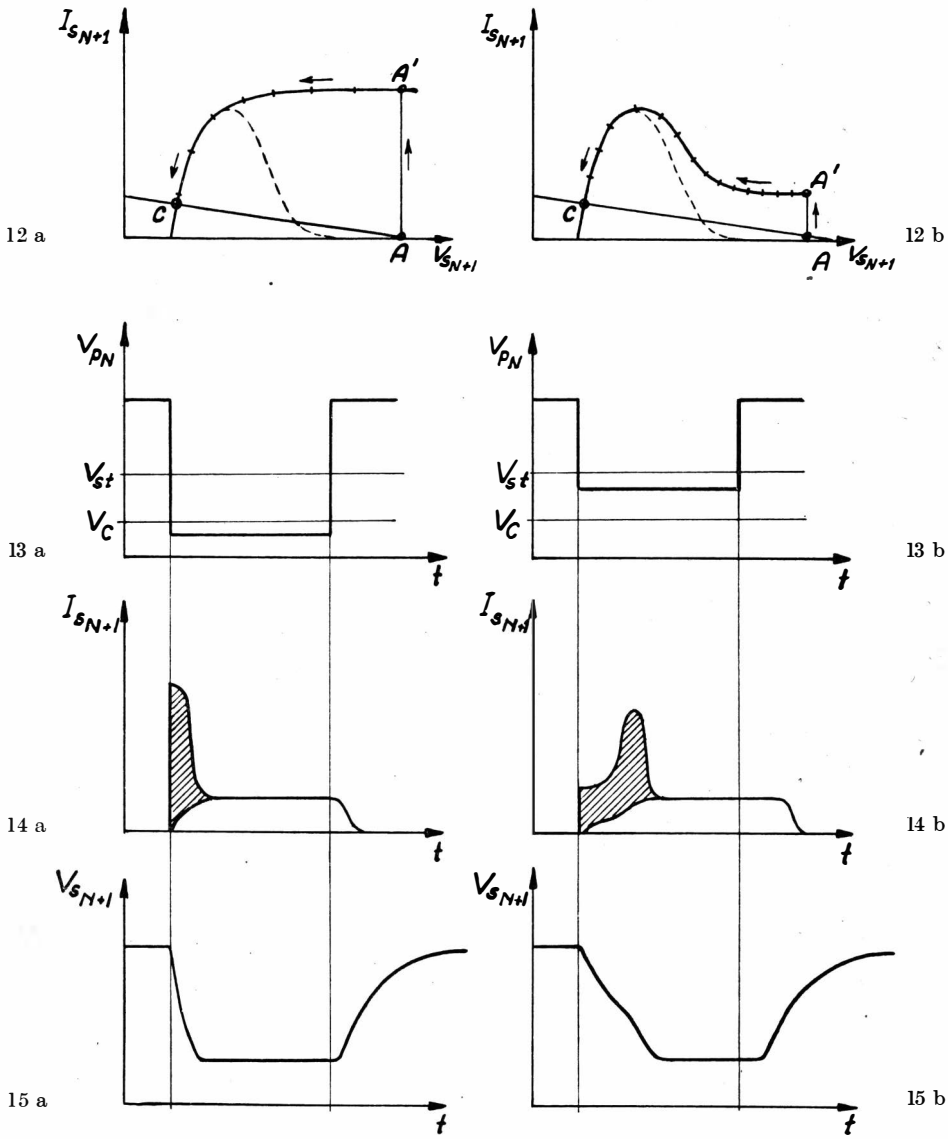


Fig. 12. Motion of the operating point during a switch-over. Fig. 13. Plate input voltage oscillogram. Fig. 14. Spade current oscillogram. Fig. 15. Spade voltage oscillogram.

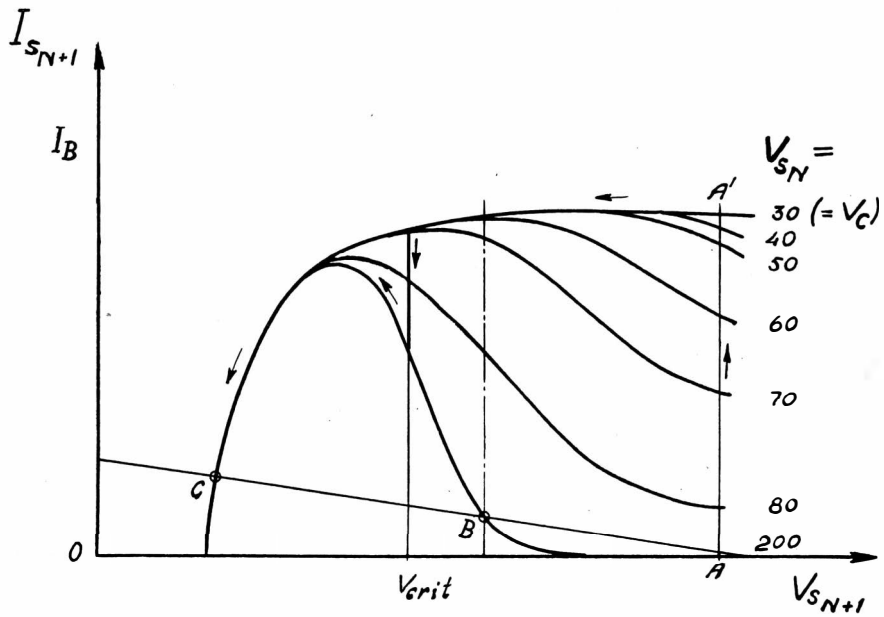


Fig. 16. Characteristics of a spade when the previous plate is at cathode voltage, for various voltages of the previous spade

$V_{s\,N-1} = V_{s\,N+2} = \ldots = V_{s\,10} = V_a = 200$ volts.

The right figures 12 b—15 b show the conditions if the plate-voltage-swing is less, that is, to say when the plate is only brought somewhat below the stepping value. The initial current to spade $N+1$ is then small, and its voltage will drop more slowly. In Fig. 12 the dots are intended as time markers, and in Fig. 14 the shadowed areas represent the charge of the spade-to-earth-capacitance, these areas being the same in both cases. Consequently, to make the switch-overs as fast as possible, the plate-voltage-swing should be extended down to, or slightly below the cathode voltage.

In the case a) above, the voltage of a spade drops more quickly than it rises, and this difference grows more predominant the larger the spade-resistances which are chosen. Thus the voltage of spade N may be considered constant at the low locking value, during the time, which spade $N+1$ needs to drop. This justifies the use of fig. 11 for studying the stepping process. (These characteristics are derived when spade N is at locking voltage V_L.)

In the case b) and also in a), if the spade-resistances are small,

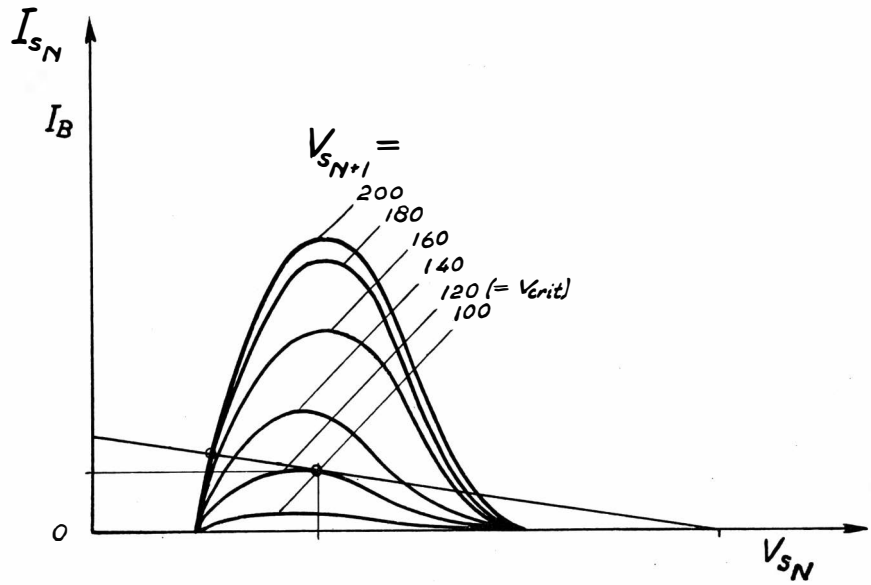


Fig. 17. Characteristics of a spade for various voltages of the following spade $V_{s\,N-1} = V_{s\,N+2} = \ldots = V_{s\,10} = V_a = 200$ volts.

the voltage of spade N may rise more rapidly than that of spade $N+1$ drops. As the capability of plate N to direct the beam against spade $N+1$ depends upon the voltage of spade N, the rise of the latter cannot be ignored. This being so, the discussion above, founded on the characteristics of Fig. 11, is at fault. As we shall find, this will give another, and more restrictive, lower limit to the spade-resistances, than that corresponding to the tangent on the top of the spade-characteristic. To get an idea of this value, we use a set of characteristics of spade $N+1$ for various voltages of spade N, Fig. 16, and a similar set of curves for spade N at various voltages of spade $N+1$, Fig. 17, both sets referring to the actual low voltage on plate N. To obtain the most unfavourable conditions, we suppose the earth capacitance of spade $N+1$ very enlarged, to make its voltage dropping-time very long compared to the time-constant of spade N. A certain voltage V_{crit}, of spade $N+1$, makes the characteristic of spade N tangential to the resistance line in Fig. 17. When this point is passed, the voltage of spade N suddenly rises to the supply value. The characteristic of spade $N+1$ then becomes shifted, the operating point in Fig. 16 is displaced downwards to the lowest

characteristic, and depending upon whether this takes place at a lower or higher voltage than that corresponding to B, spade $N+1$ either continues to decrease its voltage, or returns to the supply value. In the latter case the beam becomes diverted out of the box, and goes to the farthest end of the tube.

In this way the lower limit of the spade resistances is established. A figure of this value, however, is more easily found by an experiment with a large capacitive load on a spade, than by deriving the characteristics mentioned. Such an experiment with normal operating data shows that spade-resistances higher than twice the minimum locking-value usually secure right operation in this respect.

Small spade-resistances are also unfavourable since they reduce the plate current, partly because the spade takes more current itself and partly because the locking voltage is higher so that the spade cannot catch the whole beam but allows its lower part to pass to the farthest box. This fraction of the beam is also found in Fig. 9, showing that the current to spade 1 is not cut off before the spade N is at a very low voltage.

7. *R-C-coupled plate-pulsed circuit*

We are now ready to study the stepping process in a trochotron provided with R-C-circuits to the plates. The purpose and action of these have already been discussed in connection with Fig. 2—5. Suitable operating conditions can now be derived with the aid of Fig. 18, which shows the voltage of a plate before, when, and after the beam goes to that box. As mentioned by WALLMARK, the plate current is nearly independent of the plate voltage as long this is above the stepping value. Consequently, when the beam is locked to a box, the plate takes the excess current, and the voltage across the R-C-circuit both decreases and increases exponentially, with the time constant R_pC_p as shown in Fig. 18.

Confining ourselves at first to low and moderate pulse frequencies, we assume the exciting pulses rectangular, and of sufficient duration in relation to the switch-over time required by a spade, but short in relation to the time constant of a plate circuit. Then we choose the pulse amplitude V_i as being at least equal to the difference between the stepping voltage and the cathode voltage. (This is favourable, as appears later.) The plate supply voltage must then be sufficiently high, to prevent the plate voltage from reaching the stepping value, except when the beam has remained a sufficient

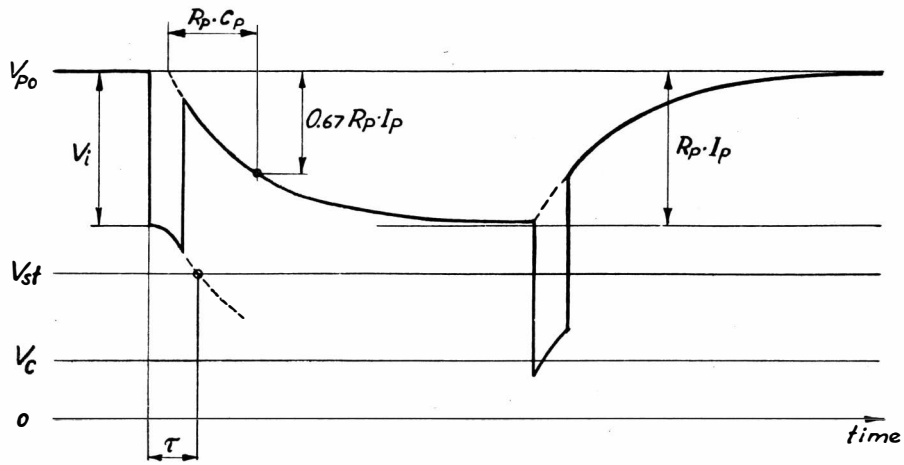


Fig. 18. The voltage of a plate in the R-C coupled-plate-pulsed circuit of fig. 2.

time in the box. The plate resistance is then so chosen, that the voltage across it, developed by the plate current, lowers its voltage almost to the stepping value. Finally, the plate capacitance is chosen, to acquire a suitable time constant. This will limit both the maximum pulse width and the maximum stepping frequency, as described below.

Considering variations in operating data, it appears from Fig. 18 that the plate supply voltage for instance may be varied within a range approximately equal, either to the pulse amplitude V_i or to the voltage drop $R_p I_p$, depending on which of these is the smallest. These limits are shown in Fig. 19 (first obtained by Hambraeus), in which the possible combinations of plate supply voltage V_{po} and pulse amplitude V_i are plotted, while all other conditions remain unchanged. However, if the circuit is to operate for small pulse amplitude, differences in the stepping voltage value among the boxes becomes important, as well as variations in the voltage drop $R_p I_p$ caused by variations in the emission. To keep the latter influence small, the plate resistances should never be chosen larger than necessary.

The maximum pulse width and minimum pulse interval (when the pulses are short) are both equal to τ in Fig. 18, since the plate voltage will reach the stepping value if a pulse exists at that moment. In fig. 19 this effect reduces the area representing right operation.

The minimum pulse width is given by the fact that the next spade

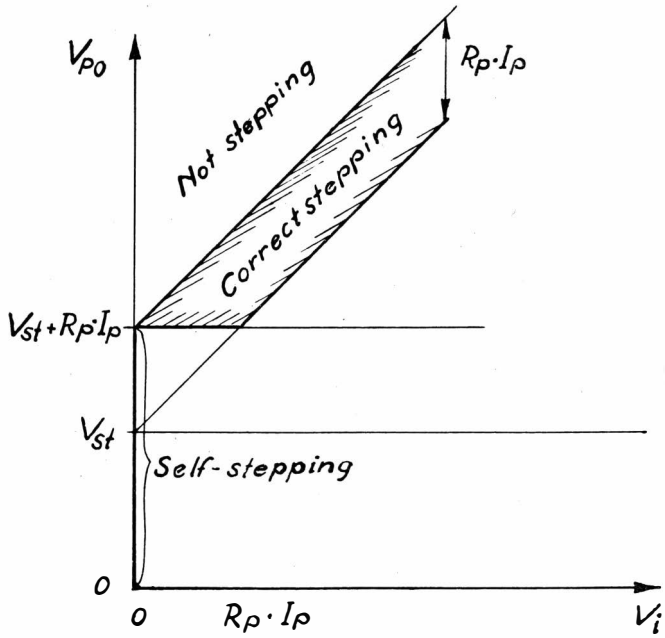


Fig. 19. Possible combinations of plate-supply voltage, V_{p0} and pulse amplitude, V_i.

must have reached a lower voltage than that corresponding to B in Fig. 11 before the pulse vanishes. As pointed out in the preceding chapter, the pulse should decrease the plate voltage below the cathode voltage in order to keep this time as short as possible.

Consequently, a high pulse amplitude is very favourable, both for fast and also for reliable operation. When we try to reach higher operating frequencies by reducing the plate-capacitances, we finally reach a point where the plate time-constants are of the same order as that of the spades, and then they grow ineffective. Consequently, we cannot reach such a high stepping frequency with this circuit as with the push-pull arrangement.

8. *Self-stepping*

In the circuit with R-C-coupled plates just described (Fig. 2), if we enlarge the plate resistances or reduce their supply voltage, so that the plate voltages can drop below the stepping value without pulses, the circuit becomes self-stepping. The beam will remain in each box for the time required by the plate to reach the stepping voltage. This time depends, amongst other things, upon the plate

supply voltage, which offers a simple method of obtaining pulse-time modulation. This will be described more fully later. (See 16.)

If the plate supply voltages are chosen below the stepping value, the plate R-C-circuits can be omitted. The beam will then step very fast, and at plate voltages below the cathode voltage it will only jump on the spades, resting on each one for only some microsecond.

9. *Other pulse-stepping methods*

In the applications it is often desirable to free the plates from the exciting pulses. As pointed out by WALLMARK, pulses could be fed to other electrodes, to this purpose, e. g. negative pulses to the spades or rail, or positive pulses to the cathode, all acting by displacing the whole beam upwards, or positive pulses to the grid or anode, acting by widening the beam. In either case the fact that the leak-current is always greatest to the next box should also be made use of. Investigations of these methods, however, have given no useful results, since they suffer from the same drawback as the basic plate-pulsed circuit described in 1. Some inherent mechanism to prevent stepping twice for one pulse must always be provided.

The by-spade trochotron, described by WALLMARK, however, seems to offer a solution. The by-spades can be connected into two groups (preferably within the tube) and fed in push-pull, acting in the same manner as the plates in (3). In such a tube, self-stepping can also be completely eliminated if the by-spade is given a suitable shape and location to take over the plate current when the plate voltage drops to a low value.

10. *Re-setting of the beam*

The simplest method to re-set the beam is to cut off the emission for a moment, as for instance, by a negative pulse on the anode or the grid.

In many applications, especially for counting purposes, it is desirable to make the beam self-restoring. Two methods of obtaining this are discussed below.

11. *Re-set by high anode-resistance*

The principle of restoring the beam by employing a high anode-resistance has already been described by ALFVÉN.

We shall now discuss the process more in detail with the aid of suitable characteristics. Referring to Fig. 20 we begin by assuming

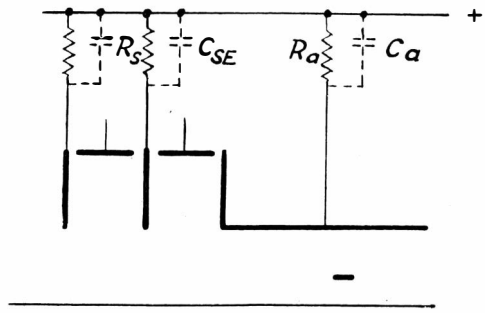


Fig. 20.

the beam locked to spade 10, with the current divided between it and plate 10, which is positive. To start the re-setting process we shift plate 10 to cathode voltage. The major part of the beam then turns to the anode according to Fig. 21, which shows the current distribution to the anode, spade 10 and spade 1, as a function of the voltage of spade 10, when plate 10 is at cathode voltage and the anode at full positive supply voltage. The current to the anode causes its voltage to drop, which diminishes the characteristics of Fig. 21, until finally the spade characteristic becomes tangential to

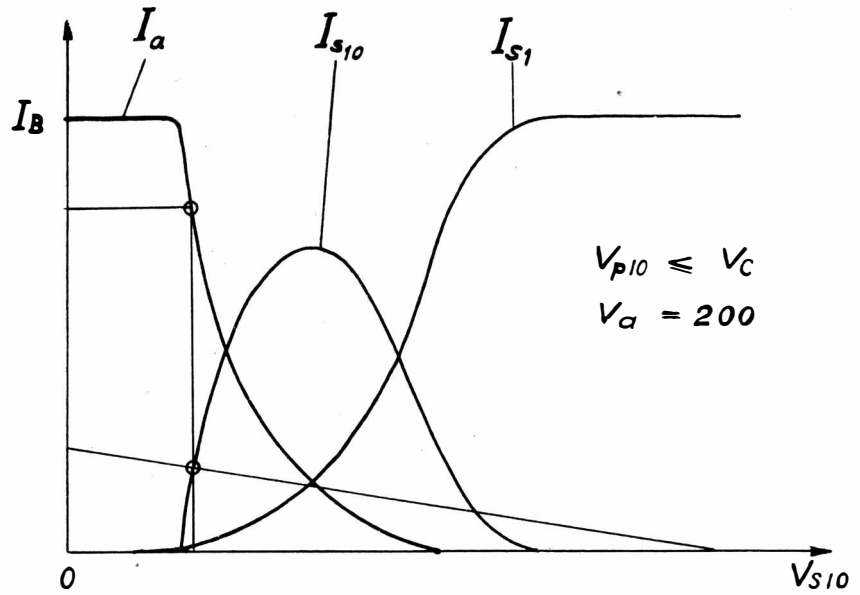


Fig. 21. Current distribution as function of the voltage of spade 10. The anode at full supply voltage.

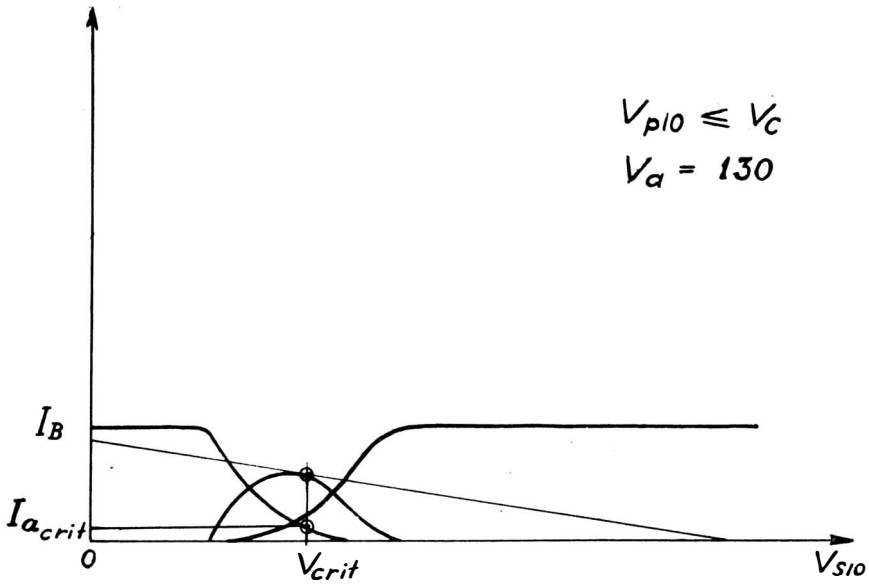


Fig. 22. Current distribution as function of the voltage of spade 10. The anode voltage reduced.

the resistance line of spade 10, as in Fig. 22, which corresponds to a certain, lower, anode voltage. After this critical stage is reached, spade 10 is able to raise its voltage, re-setting the beam to the farthest end of the tube. The anode resistance necessary, is determined by the anode voltage drop corresponding to Fig. 22, and the current to the anode corresponding to the tangential point. This will usually give a value considerably greater than a spade resistance. This however, is not sufficient, since the time constant of the anode must be great enough to keep its voltage low whilst that of spade 10 rises. This condition is usually furnished by the tube capacitances themselves, unless the spades are loaded by external capacitances. No upper limit to the anode time-constant is established by the re-set process itself, but to make the re-set rapid it should not be chosen too much greater than necessary.

12. *The re-set time*

Due to the great time-constant of the anode-circuit, a re-set takes considerably more time than a normal switch-over. The re-set gives unfavourable subsequent effects, because the beam current after returning to the farthest end of the tube, is greatly reduced and only

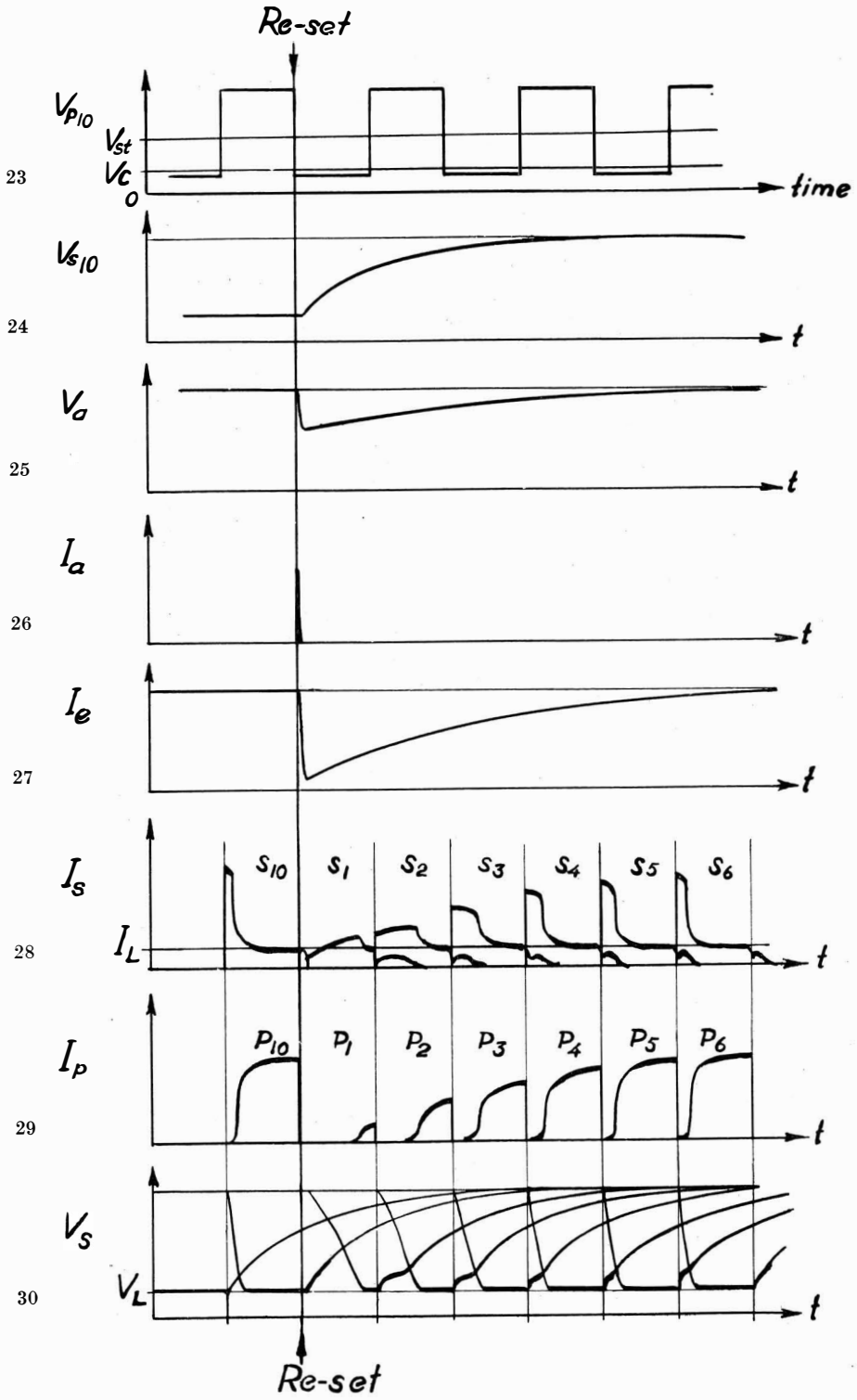


Fig. 23—30. A re-set process.

23. Voltage of plate 10. 24. Voltage of spade 10. 25. Voltage of anode. 26. Anode current. 27. Total beam current. 28. Spade currents. 29. Plate currents. 30. Spade voltages.

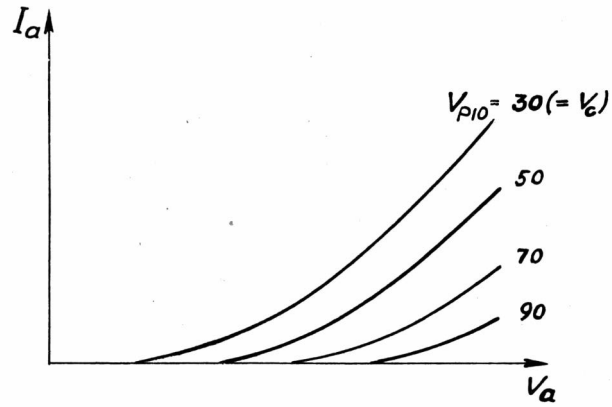


Fig. 31. Anode characteristics when spade 10 is at locking-voltage.

gradually recovers its normal value, due to the slow rise of the anode voltage. Thus, if the circuit is pulsed to step at a fast rate, this rate will be essentially limited by the great switch-over-times required by the first boxes. Fig. 23—30 are intended to illustrate the oscillograms of voltages and currents in a circuit operated near the upper frequency limit. When the plates are excited in push-pull, the fall time of the first spade must be within one pulse interval, but if the plates are supplied with R-C-circuits, the plate currents must also be utilized, and might not be too different in the first boxes, thus establishing an upper frequency limit essentially lower than for the push-pull circuit. The frequency limit however, can be a little extended if the first spade is permanently kept at cathode voltage. Then the second spade limits the pulse interval, or the minimum pulse width, respectively.

We have hitherto assumed the voltage of plate 10 as being brought down as far as, or even below, the cathode voltage. If this is not the case, the conditions becomes more unfavourable, the anode resistance must be further enlarged, and can, as pointed out by ÅSTRÖM, be obtained from Fig. 31, the anode characteristics, when spade 10 is at the critical voltage, corresponding to the tangential point of Fig. 22, and for various voltages of plate 10. Consequently, it is very essential to drive plate 10 to a low voltage in order to secure reliable operation.

13. *Re-setting by a special electrode*

To overcome the disadvantages of the method just described, BJÖRKMAN has suggested a special electrode to divert the beam from

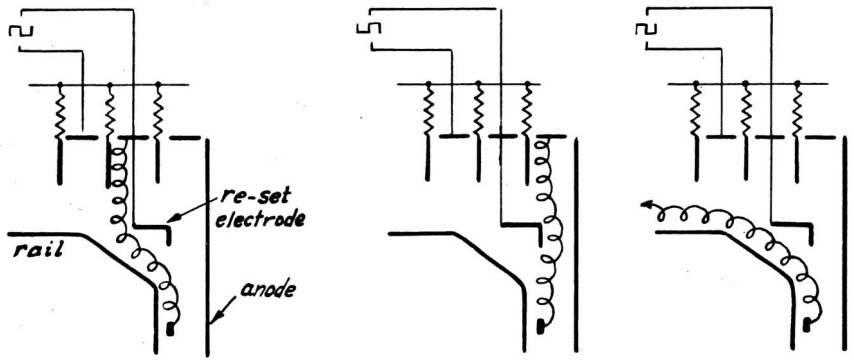


Fig. 32 Fig. 33 Fig. 34

Fig. 32—34. Re-setting by a special electrode.

spade 10. This arrangement is shown by Fig. 32. The extra electrode is connected to plate 9 and its voltage shifted between a positive value, and zero. Thus, the beam travels alternately above and below this electrode, but this is of no importance except when it goes to box 10. We begin by assuming the beam locked to spade 9, as in Fig. 32. When plate 9 and the re-set-electrode shifts down, it becomes locked to spade 10 instead, see fig. 33. Finally, when the re-set electrode is raised in voltage, the beam passes below it to the far end of the tube, screened from the voltage of spade 10, as shown by Fig. 34.

Applications

14. *A pulse counter*

A scale, specially intended for the counting of Geiger-Müller-pulses has been constructed, with a ten-box trochotron as the counting element. The push-pull-drift is utilized, as shown by the wiring diagram in fig. 35. The pulses to be counted are fed via an amplifier tube with limiting and pulse-standardizing action, to a conventional »scale of two», from which the anodes are connected to the plate-groups of the trochotron. Thus every input pulse displaces the trochoidal beam one box. At every tenth pulse, when the beam is re-set, a negative pulse from the first spade goes to a mechanical counter via a trigger.

The beam position in the trochotron is indicated by small glow-lamps connected to the spades.

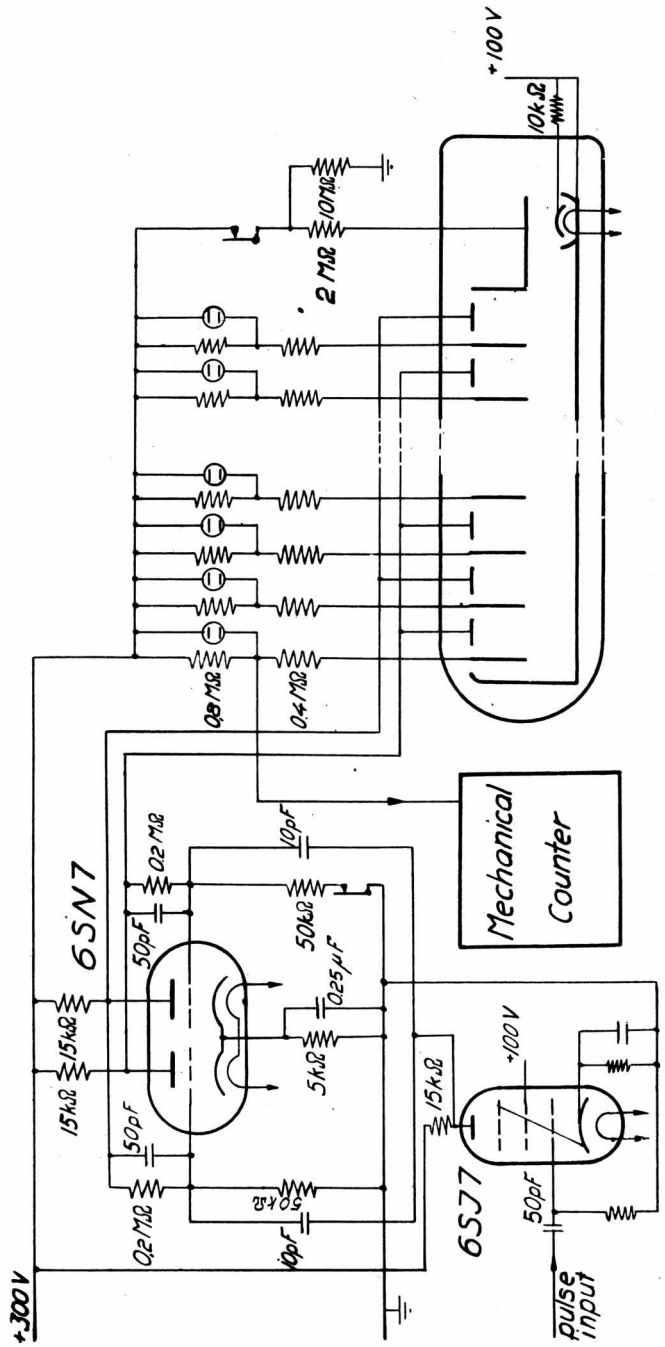


Fig. 35. Wiring diagram of a pulse counter.

The operating data of the trochotron are here:

Magnetic field strength	240 gauss
Anode, spade and plate supply voltage	200 volts
Cathode voltage, developed in the cathode resistor, about	30 volts
Beam current (emission)	0.8 mA
Tolerances in supply voltages (all varied simultaneously) better than	±20 %
The tolerances on spade resistors are very great (more than	±20 %)

The trochotron in this unit is able to count slightly more than 100 000 pulses per second, i. e. the resolving time is better than 10 microseconds. As the mechanical counter is very much slower, the unit could with advantage be supplied with another trochotron, counting every tenth pulse, and giving every hundredth to the mechanical counter.

In order to set the counter at zero before use, not only the trochotron but also the »scale-of-two» must be restored. This is achieved by a switch, which breaks both the anode supply voltage to the trochotron, and one grid resistor connection to the double triode, as seen from fig. 35.

15. *A chronoscope*

A short-time measuring equipment, utilizing ten-box trochotrons as frequency dividers, has been designed by LUNDQUIST and ÅSTRÖM. The principle is shown in the block-diagram of fig. 36. From an oscillator of high frequency accuracy, pulses are taken out through a gate, during the time interval to be measured, and are counted by the trochotrons. The first trochotron is stepped directly by the pulses from the gate, and from it every tenth pulse continues to the next trochotron and so on until the frequency is low enough to operate a mechanical counter. After a measurement is taken, the positions of the trochotron beams will show tenthousandths, thousandths, and hundredths of seconds, while the mechanical counter shows tenths, units, tens, etc. Thus the time is given in seconds and decimals of seconds, with an accuracy determined only by the oscillator.

For interchangability, all the dividers are designed with the same wiring diagram, fig. 37, the plate-pulsed method being employed.

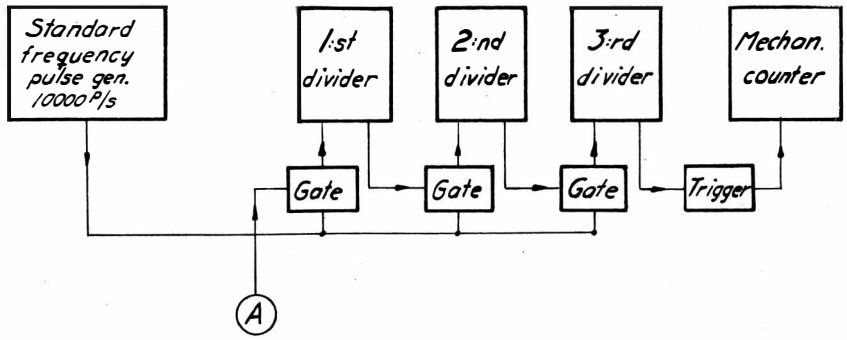


Fig. 36. Block-diagram of a chronoscope. A: Input of pulse to be measured.

The glow-lamps indicating the beam positions are here connected to the plates parallell to the R-C-circuits. The action of these is thus a little affected. The voltage across one of them, when the beam goes to the plate, is constant, equal to the operating voltage of the glow-lamp, V_g. The plate resistance is designed to carry the surplus plate current and also serves to discharge the plate capacitance. The oscillogram of the plate voltage is shown by fig. 38. As all the stages have equal plate circuits, they must be fed with equal pulses derived from the standard pulse generator. To this purpose every stage is equipped with a pulse-input gate which is opened by a positive pulse from the previous stage every time this is re-set.

Before a measurement is taken, all units should be set at zero. This is performed by switching down the anode supply-voltages of the trochotrons for a moment.

Investigation of a complete stage showed that the operating data could be varied within the following limits:

Anode supply voltage	175—210 volts
Spade » »	150—200 »
Plate » »	175—225 »
Pulse amplitude	75—100 »
Emission	0.65— 1.0 mA
All voltages simultaneously	±15 %

The magnetic field was then kept constant at 240 gauss, the pulse width 10 μs and frequency 10 000 pulses p/s. It could, however, operate until 30 000 pulses p/s.

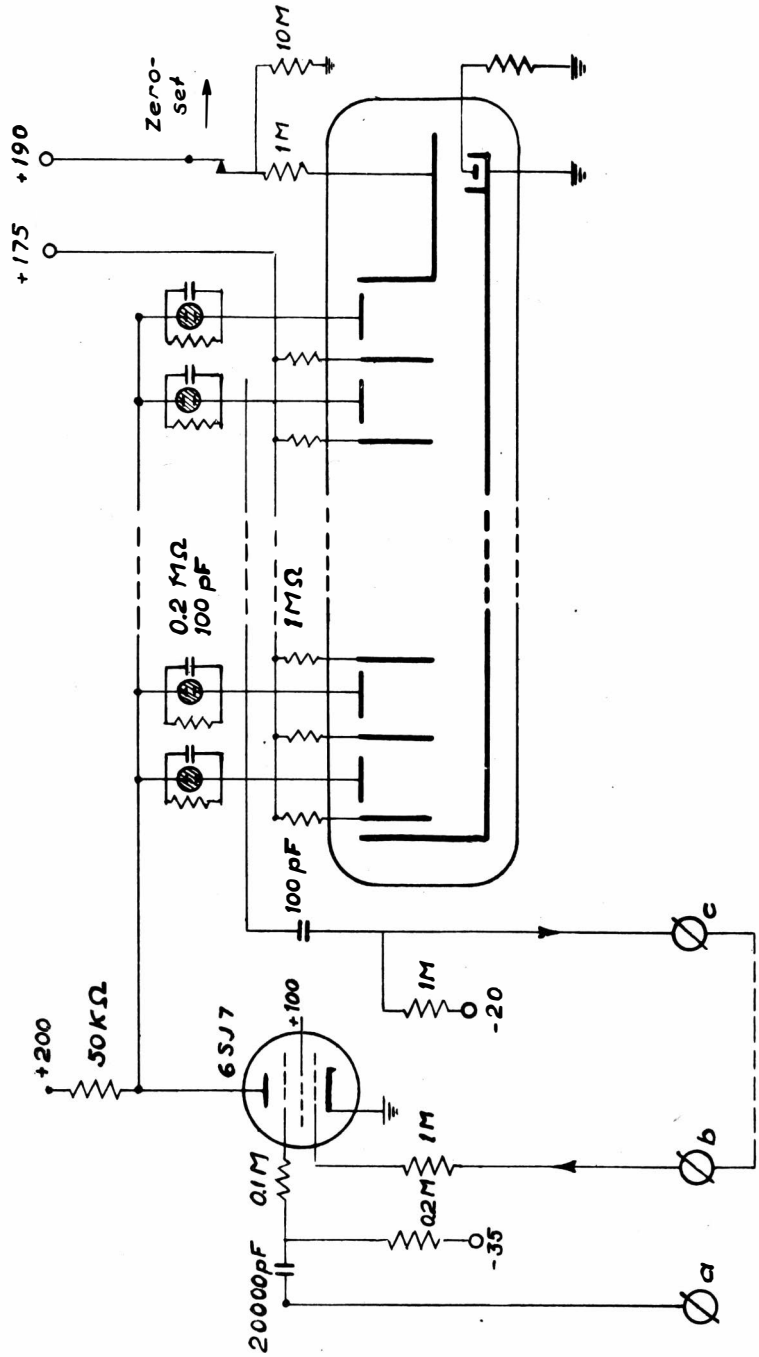


Fig. 37. Wiring diagram of one chronoscope-stage.
a. Input of standard pulses.

The chronoscope could also be provided with stages measuring tens of microseconds and probably also units, but in this case the push-pull-coupled driving method should be employed.

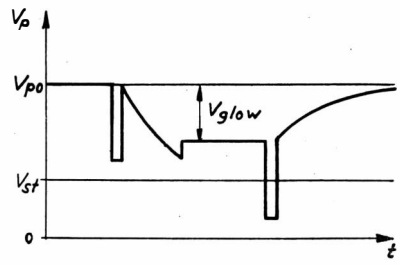


Fig. 38. Voltage-oscillogram of a plate in the chronoscope.

Pulse-time-modulation by trochotrons

The trochotron is well suited to multi-channel telephony systems, working with pulse-time-modulation. Some circuits will be described in which the trochotron acts both as a cyclic switch and modulator, respectively de-modulator. The results here presented are a summary of investigations by L. LINDBERG, S. WARRING and E. ÅSTRÖM.

16. *Circuits based on stepping trochotrons*

In the chapter on pulse-technique it is described how the beam becomes self-stepping when the plate supply voltage is low. See (8). This gives a method of easy achieving time-modulation, since the beam will remain in a box for a time which is almost a linear function of the plate supply voltage in that box. Every time the beam steps, a sharp voltage pulse may be taken from the next spade to be amplified and transmitted. On the receiver side, these pulses are used to make a trochotron stepping synchronised with the transmitting one. The plates in that trochotron thus receive width-modulated charge-pulses which are easily de-modulated by integrating circuits.

17. *Modulator circuit*

A basic modulator circuit is shown in fig. 39. For the sake of simplicity, only three channels are shown. The odd plates, utilized for modulation are furnished with high-resistive R-C-circuits. The even boxes serve as resting places for the beam.

Suppose the beam has just arrived at box 1, at zero time. The plate voltage there, initially equal to the supply value, begins to drop according to fig. 40, until the stepping voltage is reached at the time t_o. At this point the voltage of spade 2 suddenly drops and switches the beam over to box 2. If the supply voltage of plate 1 is increased or decreased, due to modulation, the time t_o becomes

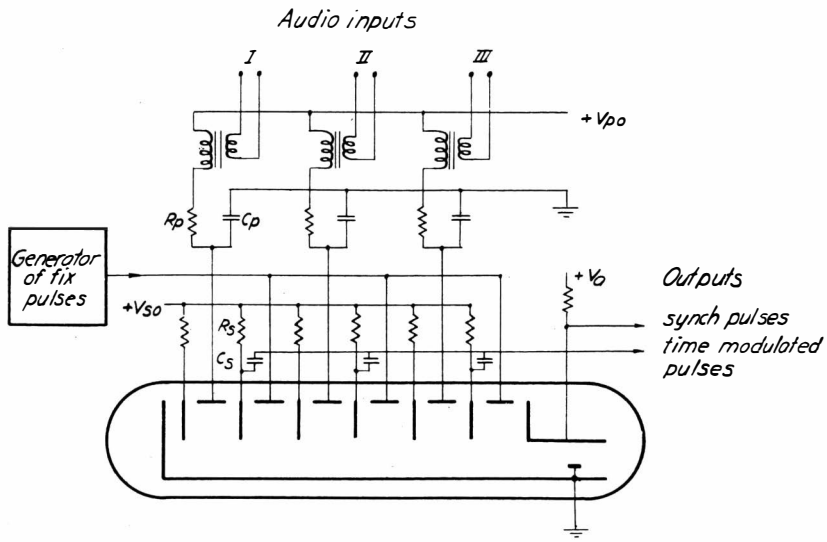


Fig. 39. Stepping modulator circuit for pulse-time-modulation.

modulated as is also shown in fig. 40. When the voltage of spade 2 drops, a sharp negative voltage pulse goes through the small capacitive coupling C_s to the pulse-output line. The beam, having entered box 2, must be kept there until a fixed moment before it is utilized by channel 2. To this purpose the even plates are normally kept positive, and fed by fixed negative pulses from an external generator. Consequently the beam steps alternately at modulated and at fixed time-points along the tube, until it reaches the anode, where it is re-set by the high anode-resistance. From this, a pulse may be taken out and used for synchronizing purpose. The current distribution in time is shown by fig. 41, where the shaded areas represent the modulation ranges.

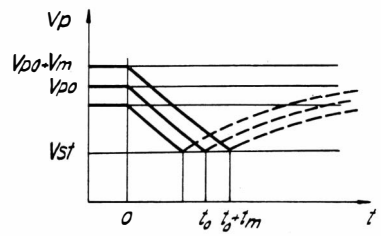


Fig. 40. Principle of achieving time-modulation by varying the supply voltage of a plate.

18. *De-modulation*

A basic circuit is shown by fig. 42. In this trochotron the beam should be shifted in exact synchronism with that in the modulator tube. To this purpose the odd plates are fed with the received time-modulated pulses, and the even plates by fixed pulses synchronous

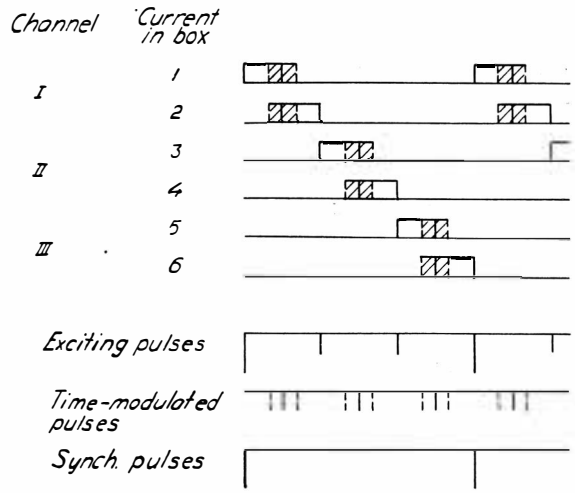


Fig. 41. Distribution of current and pulses in time.

with those used by the modulator. The anode is fed by the synchronizing pulse. The current distribution in time is the same as in fig 41. Here the currents to the rest-boxes are also utilized, to obtain outputs in push-pull.

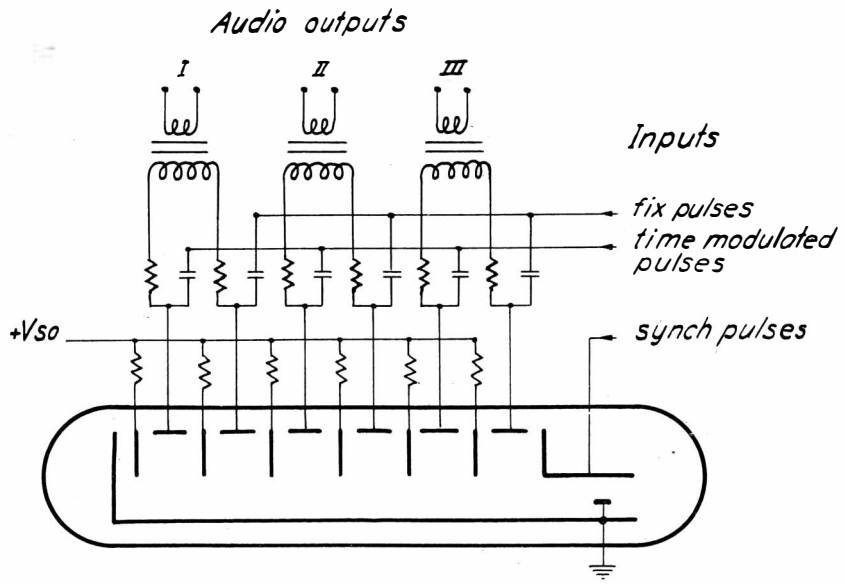


Fig. 42. Pulse-stepping de-modulator circuit.

19. *Power, linearity, noise and cross-talk*

Circuits for telephony make very high demands on the trochotrons. At first hand the precautions mentioned in the chapter on pulse technique must be taken, but attention must also be paid to sources of cross-talk and noise. We shall restrict ourselves here to a few of them, which are inherently associated with the trochotron.

The specific trochotron noise (see ÅSTRÖM *loc. cit.*) is overcome by keeping the beam current within a safe value.

Cross-talk is caused by capacitive effects within the tube, but also by »leak-currents». These are of three types: Firstly, leak currents above the beam caused by its degeneration. These are essentially picked up by spades, and do scarcely any damage. Secondly, leak currents escaping through interstices between electrodes and insulation. These are picked up by lead-in wires and cause complicated effects, which however can be kept of minor importance by careful designing of the tube. Thirdly, leakage current continuing below a spade due to its inability to catch the whole beam. This current goes entirely to the farthest end of the tube and may be picked up there by an extra electrode.

As far as the power is concerned, the modulator requires much less than can be supplied by an ordinary carbon microphone, but at a higher voltage, making a step-up-transformer necessary. The de-modulator may deliver sufficient output power, at least if the channels are few. Consequently, audio amplifiers in every channel are unnecessary, at least at the modulator.

The linearity would be nearly exact if the plate resistances at the modulator could be chosen infinitely great. However, the time constant of the plate-circuits must be kept sufficiently small, to avoid cutting off the high modulating frequencies.

20. *Circuits based on cathode-voltage-switched trochotrons*

Pulse-time modulation can also be acquired by another method of operating the trochotron, in which the beam is not stepping but is shifted by the cathode voltage.

Referring to fig. 43 we assume all spades and plates as being connected to an ideal voltage divider. The cathode, as well as anode and rail, is supplied by a variable voltage V_c. When we change V_c, we get a current distribution to the different electrodes similar to fig. 44.

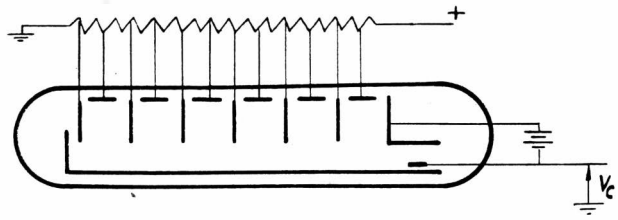


Fig. 43. Cathode-voltage-switched trochotron.

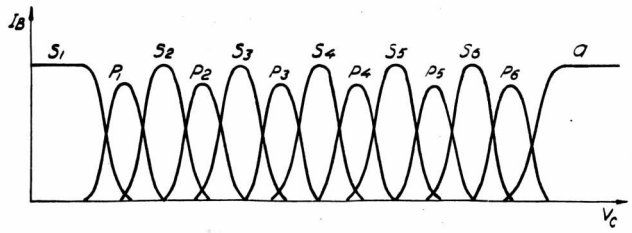


Fig. 44. Current distribution as function of the cathode voltage.

21. *Modulation*

We now suppose the cathode fed by a saw-tooth voltage of the cycling frequency and an amplitude equal to the voltage between the first and last spades, as shown by fig. 45. Restricting ourselves to one box, if we superimpose a modulation on the plate voltage, the current distribution in that box becomes affected, according to fig. 46. That is, when the voltage of plate N is increased, the current to spade $N+1$ arrives later (not until at a higher cathode voltage) than normally. In this way the rising slope of the current curve for the next spade becomes modulated by the plate voltage. The current to the preceding spade N also becomes somewhat modulated, but only on its falling slope. This can be understood by the fact, that the rising slope of the current curve of spade N is produced when the beam goes to box N—1, where it is shielded from influence from plate N.

A complete modulator circuit is seen in fig. 47. In order to convert the time-modulated spade-currents into voltage pulses, a small resistance is inserted in series with every spade. The voltages, developed across these, are taken out through small capacitances with differentiating action to the common output line, for further amplification to operate a trigger. In this circuit it is evidently sufficient

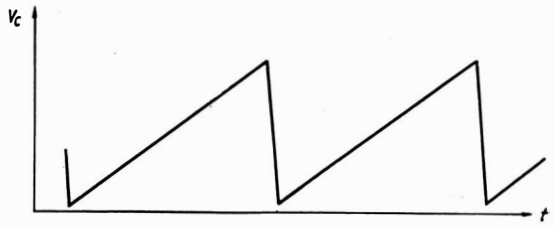


Fig. 45. Cathode voltage.

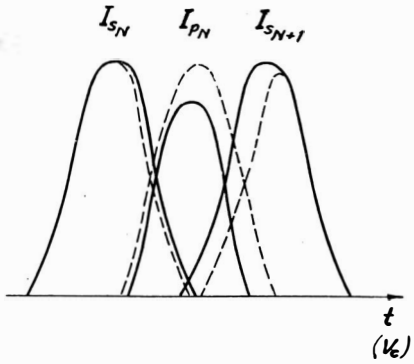


Fig. 46. Principle of time-modulation.

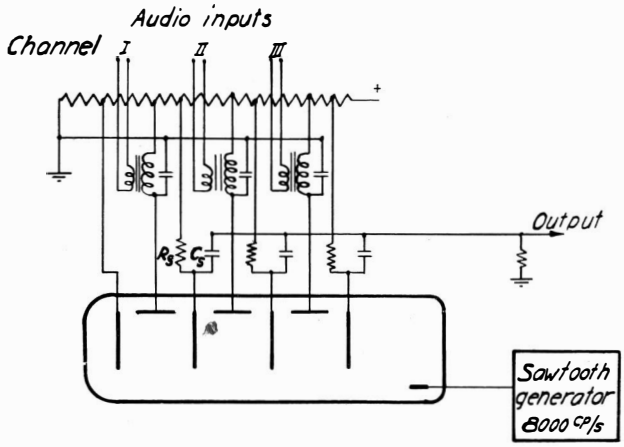


Fig. 47. Cathode-voltage-switched modulator.

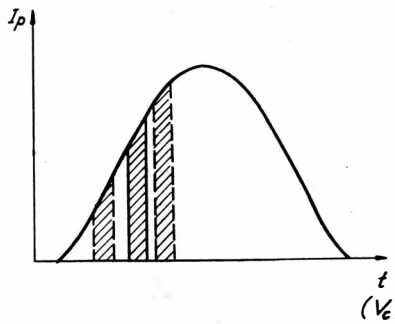


Fig. 48. De-modulation.

to provide each channel with only one box. However, the output from the trochotron is very small and requires rather high amplification.

22. *De-modulation*

The cathode-voltage-switched trochotron is also suitable for de-modulation. The circuit is similar to fig. 47, but with the spade resistances omitted. The received time-modulated pulses are used to put on the emission for a short moment each, and the slopes of the plate-current curves of fig. 44 are used for de-modulation as is evident from fig. 48. The out-put power, however, will be very small.

23. *Experimental results*

Some preliminary investigations were carried out in this laboratory at an early stage of the development of the trochotron. In spite of this, the results were quite promising. With a three-channel system of the previously described, stepping type, communication could be achieved without any audio amplifiers in the channels. The cathode-switched circuits were also investigated and found to work satisfactorily. With a combination of a modulator of the latter type and a stepping de-modulator the cross-talk could be reduced below 60 dB.

Acknowledgement

The author wishes to thank prof. HANNES ALFVÉN sincerely for his great interest and most valuable advice during the work.

Finally I would also like to thank all the collaborators for a good team-work.

Electronics Laboratory.
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Stockholm.

EXPERIMENTAL INVESTIGATION ON AN ELECTRON GAS IN A MAGNETIC FIELD

BY

ERNST O. ÅSTRÖM

I. Current to a negative electrode

1. *Earlier experiments*

Theory and experiments agree well for electron tubes when no magnetic field is present. If, however, a magnetic field is present, essential divergencies appear. In a cylindrical magnetron the anode current should cease abruptly and completely at a certain critical magnetic field strength, as shown by HULL.[1]) The experimental cut-off curves have rounded edges and the current does not drop to zero above cut-off. Further, the cut-off occurs at a magnetic field strength up to ten per cent greater than the expected value.[2])

Some years after HULL's experiment, LANGMUIR[3]) found that in the cylindrical, non-oscillating magnetron there were electrons having sufficient energy to go to the end plates, which were negative relative to the cathode, and WIGDORTSCHIK[4]) has shown that the velocity obeyed the MAXWELLIAN distribution law.

LINDER[5]) has also found a MAXWELLIAN distribution, and shows that the mean velocity is directly proportional to the square root

[1]) HULL, W. A. The effect of a Magnetic Field on the Motion of Electrons between Coaxial Cylinders. Phys. Rev. vol 18 p 13 1921.

[2]) HARVEY, A. F. The Cut-Off Characteristic of Single Anode Magnetron. Proc. Camb. Phil. Soc. vol 35 p 637 1939.

[3]) LANGMUIR, I. Scattering of Electrons in Ionized Gases. Phys. Rev. vol 26 p 585 1925.

[4]) WIGDORTSCHIK, I. M. Die Geschwindigkeitverteilung der Electronen unter dem Einfluss eines magnetischen Feldes im Hochvacuum. Phys. Zeitschr. d. Sowjet. vol 10 p 245 1936.

[5]) LINDER, E. G. Excess-Energy Electron Motion in High-Vacuum Tubes. Proc. Inst. Rad. Eng. vol 26 p 346 1938

of the anode voltage when the radius of the electron path is unchanged and the emission is so large as to give space charge limitation. Further, it is inversely proportional to the square root of the magnetic field strength if the anode voltage is constant.

During experiments in this laboratory HERNQVIST[1]) found that in the »trochotrons» too, currents appear at negative electrodes and WALLMARK and STRÖMBERG[2]) have further studied this phenomenon.

It is clear that there is some mechanism which changes the energy of the individual electrons. All the above mentioned authors agree that the tube is really non-oscillating, but a satisfactory explanation is not to be found in literature on the subject. MALMFORS,[3]) however, has recently developed an interesting theory.

2. *Measurements on trochotrons*

In the cylindrical magnetron the electron stays for a long time in the space near the cathode before they finally strike one of the electrodes, while in the trochotron the drift causes the electrons to leave the cathode region. In the magnetron one can only estimate the length of time an electron has been in the electron gas, as has been done by LINDER,[4]) while in the trochotron, the length of time an electron has been in the electron gas when it is on the distance l from the cathode can easily be computed.

In the magnetron the size of the electron path is determined by the applied voltage and the magnetic field strength. In the trochotron it is possible to change the size of the path without changing the field strength, if an electron gun is introduced.

3. *The tubes and circuit used*

In figure 1 the type of trochotron used in this investigation is shown.

The cathode was either of tungsten or a band of nickel, the latter coated with oxide. The cathode was situated about midway between the anode and the rail.

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[1]) HERNQVIST, K. G. Investigation on Electron tubes with Trochoidal Electron Paths. (Unpublished.)

[2]) STRÖMBERG, B. Investigation on the Current to Negative Electrodes in the Trochotron. (Unpublished.)

[3]) MALMFORS, K. G. On the Instability of an Electron Gas in a Magnetic Field.

[4]) LINDER. *Loc. cit.*

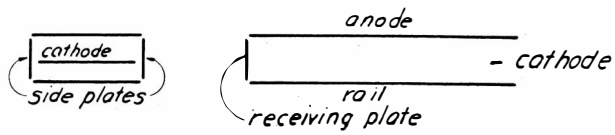

Fig. 1. The type of trochotron used in this investigation. (For further information see table at the end of this paper.)

In all but one of the used tubes, the rail and the side plates are connected together and when speaking of the rail in these cases, we mean this composite electrode.

The trochotron was placed in a homogeneous magnetic field, perpendicular to the side plates. An electro-magnet was used.

Trochotrons very easily produce parasitic oscillations, and the more numerous the electrodes are the greater is the difficulty in stopping these oscillations. It has been found necessary to use damping resistors, 100 à 1 000 ohms, decoupled by capacitors, 10 000 $\mu\mu F$. During present experiments only the leads to the filament did not contain such resistors.

The anode was positive, and the magnetic field always of such value that the electrons followed trochoidal paths to the receiving plate, which was also positive.

4. *Definitions*

The electron current which flows to negative electrodes in a non-oscillating high vacuum tube we shall call »N-current» in order to distinguish it from currents coming from other causes.

Throughout this investigation, all voltages are counted relative to the »rail». The anode voltage then defines the electric field strength in the space remote from the cathode, and the cathode potential the least energy (in electron volts) an electron must gain to be able to go to the rail.

5. *Measurements*

a) N-current to a segment of the rail

In the tube used in this experiment, see fig. 2, the rail was divided into 17 segments, of which 16 were 5 mm long (in the direction of

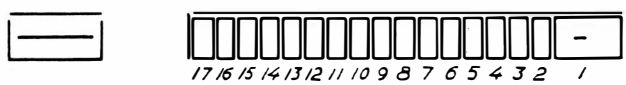

Fig. 2. Tube J15 with the rail (and side plates) divided in segments.

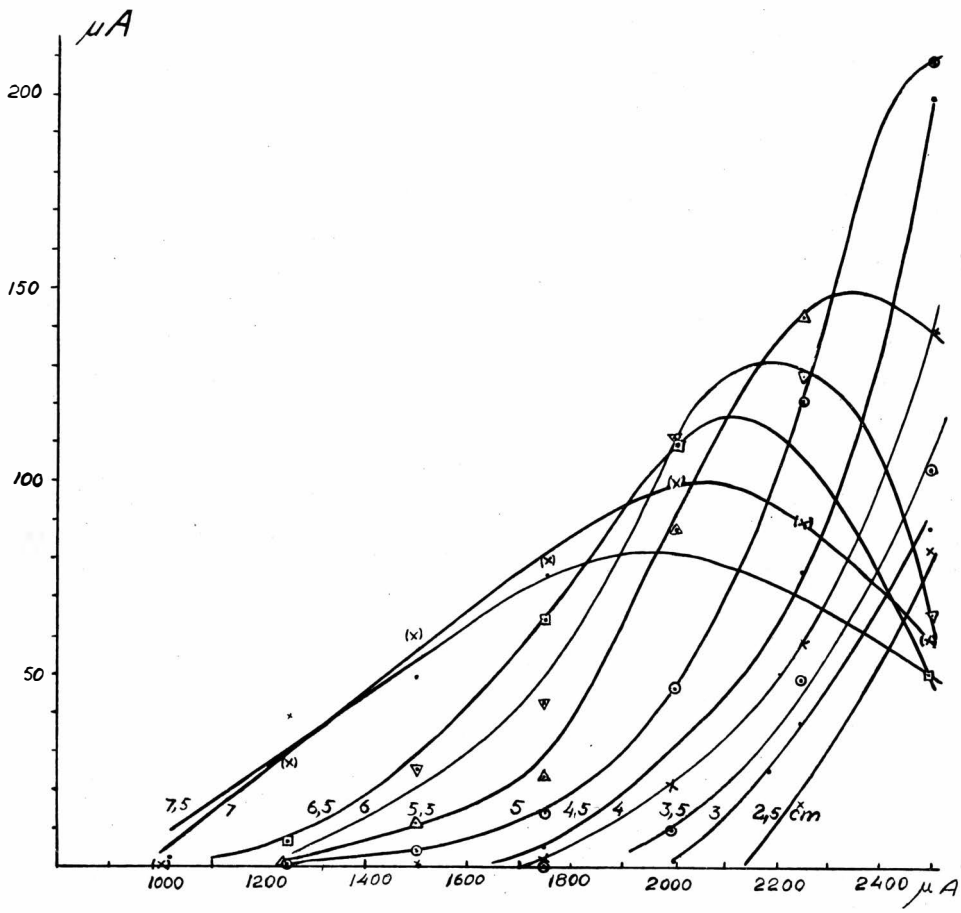

Fig. 3. N-current to a segment as a function of emission with the distance to the cathode as a parameter.

Tube J15	
Anode voltage	400 Volts
Cathode voltage	0 Volt
Magnetic induction	640 Gauss

the tube length) and the 17th longer, and placed under the cathode (cp. a table at the end of this paper). The cathode was situated about 5 mm from the nearest of the smaller segments. The segments were numbered, with number one under the cathode and with increasing numbers towards the »receiving plate».

The whole rail was at zero potential. The N-current to a certain segment was determined as a function of the emission, the latter

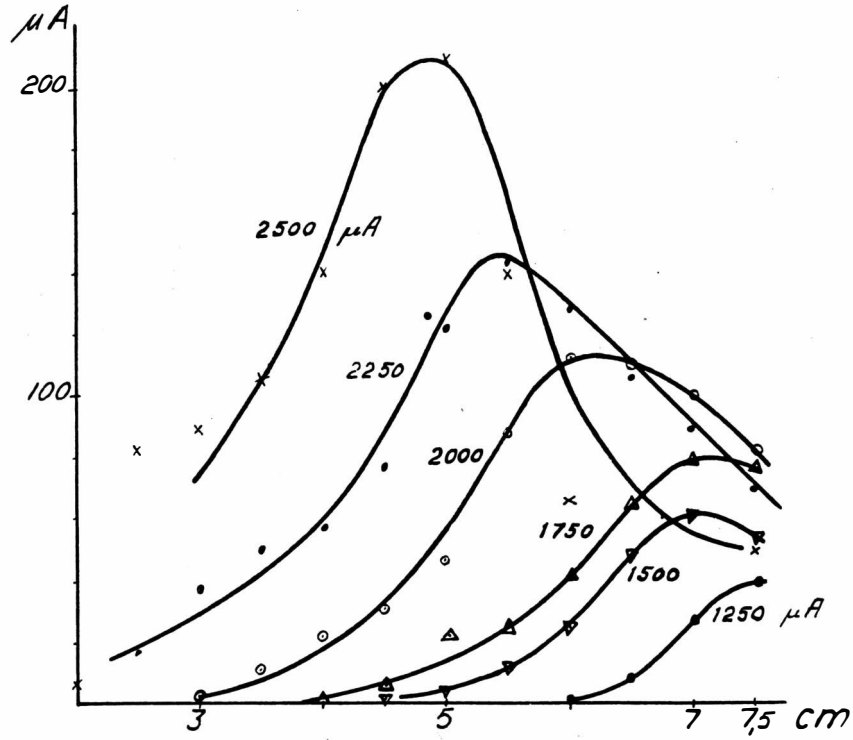

Fig. 4. N-current to a segment as a function of its distance to the cathode with the emission as a parameter. Obtained from the same measurements as fig. 3.

being adjusted by alteration of the cathode temperature. The measurement was repeated for each of the other segments. The current to number 16 and 17 was not measured, since these segments were situated near the receiving plate, which disturbed the electric field, so that measurements on these segments could not be compared with the measurements on the remaining ones.

In figure 3 we reproduce the result of such an experiment. The N-current first increases with the emission, reaches a maximum, and then decreases fairly slowly. The greater the distance from the segment to the cathode, the smaller the maximum N-current, and the further it is displaced towards smaller emission.

When, on the other hand, we keep the emission constant and measure the N-current to each of the segments, we find, see figure 4, that the N-current increases with the distance from the cathode, reaches a maximum, and then decreases. The maximum N-current

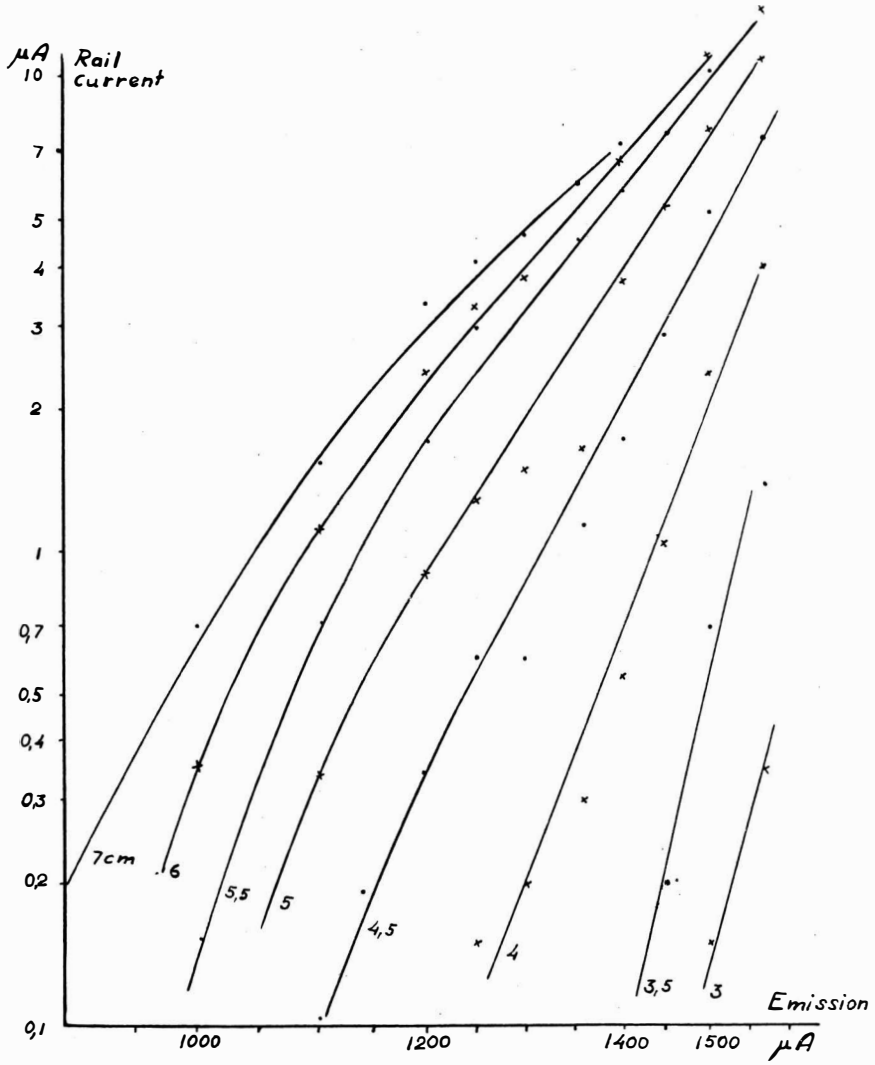

Fig. 5. *N*-current to a segment as a function of emission with the distance to the cathode as a parameter. Logaritmic axis.

Tube J15	
Anode voltage	400 Volts
Cathode voltage	100 Volts
Magnetic induction	1 130 Gauss

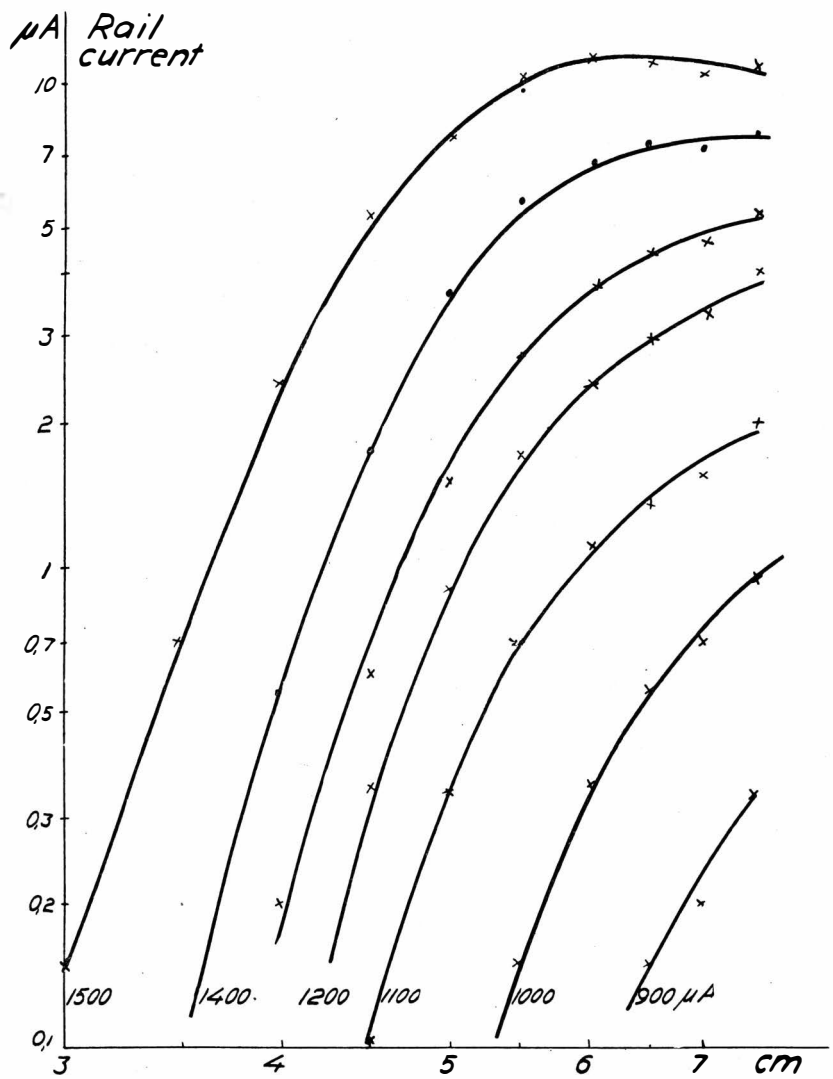

Fig. 6. N-current to a segment as a function of its distance to the cathode with the emission as a parameter. Obtained from the same measurements as fig. 5.

increases more than linearly with emission. Further, the maximum is displaced towards the cathode for increasing emission.

During the last experiment the cathode was on zero potential. By increasing it to a fourth of the anode voltage the N-current decreased and we obtained the curves in figure 5 and 6.

From the former figure it follows that when the emission is small

the N-current to a segment increases very rapidly with emission, and from figure 6 we can see that the N-current increases very rapidly with the distance from the cathode before the maximum is reached.

Since a few segments near the cathode do not obtain any considerable amount of N-current, as do all the following, it is evident that the process causing the N-current takes place in the electron gas when it is drifting along the tube. Not at all or only partially can some process near the cathode be responsible for the N-current.

b) N-current to the rail as a function of the emission.

By integrating one curve in figure 4 from zero to distance l from the cathode we obtain the current to this part of the rail. By doing the same with the other curves in the same figure we get the N-current to the rail as a function of emission, and can plot a curve of the type found in figure 7. This one, however, has been obtained by directly measuring the current to all segments, except the one nearest the receiving plate, for various values of the emission.

At low emission the ratio of N-current to emission seems to increase as rapidly as the N-current to a single segment, but at high emission the increase is much slower, which is necessary, as the N-current cannot exceed the total emission.

With 75 volts on the cathode, 8 per cent of the emission, 2 500 μA, appears as N-current, and at 50 volts it has increased to 20 per cent. A surprisingly large part of the electrons have thus gained a considerable amount of energy.

c) Relation between tube length and emission for I_r/I_e = constant

During this experiment we first measured the current to segment one. The cathode temperature, and accordingly the emission, was increased until a certain arbitrary percentage N-current to this segment was obtained. Then the experiment was repeated, when both segment 1 and 2 were connected to the instrument, measuring the N-current. The part of the rail was in the first case 5 mm and in the second case 10 mm, and for each segment then connected to the previous ones the regarded part of the rail increased by 5 mm.

When doing so we found that for a certain arbitrary percentage N-current the emission must be decreased when increasing the length of the rail, see figure 8.

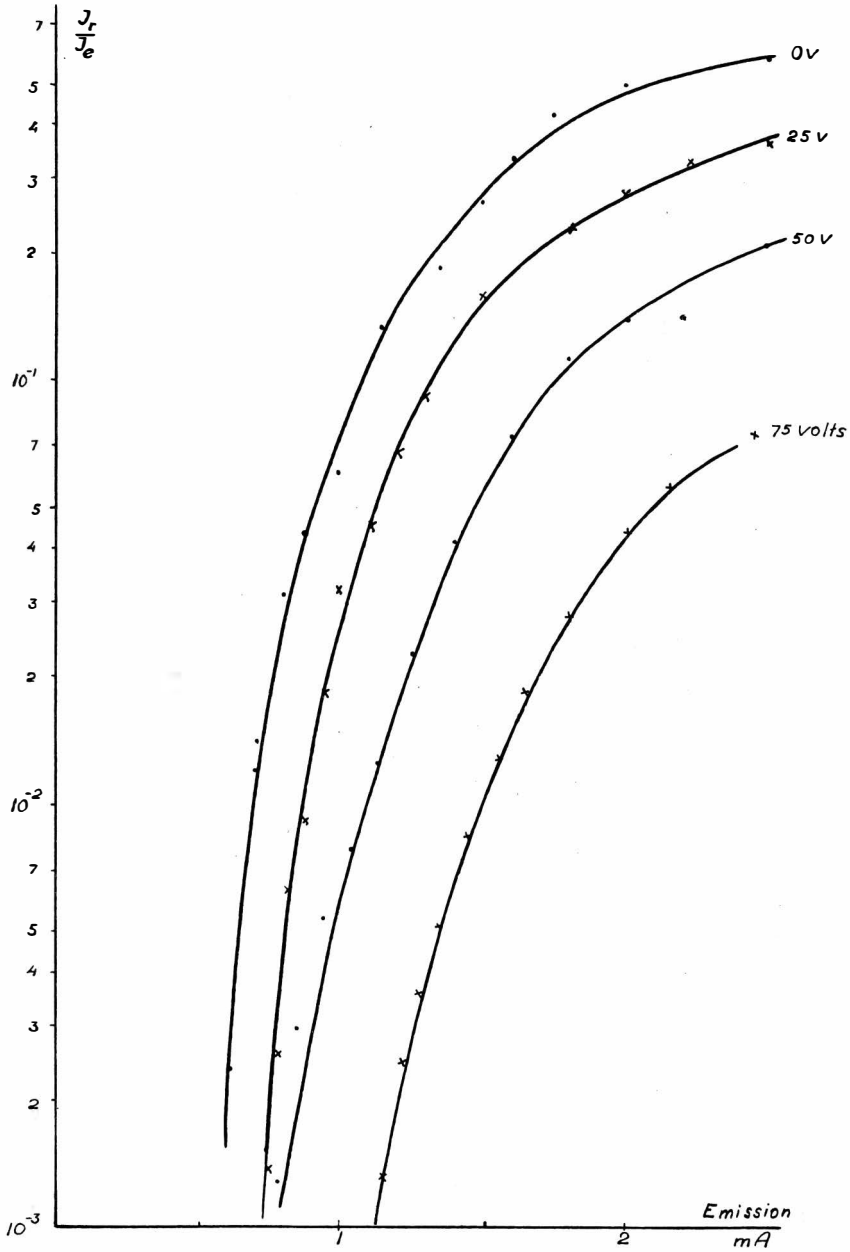

Fig. 7. The rail current as a function of emission, with the cathode voltage as parameter.

Tube J15
Anode voltage 400 Volts
Magnetic induction 640 Gauss

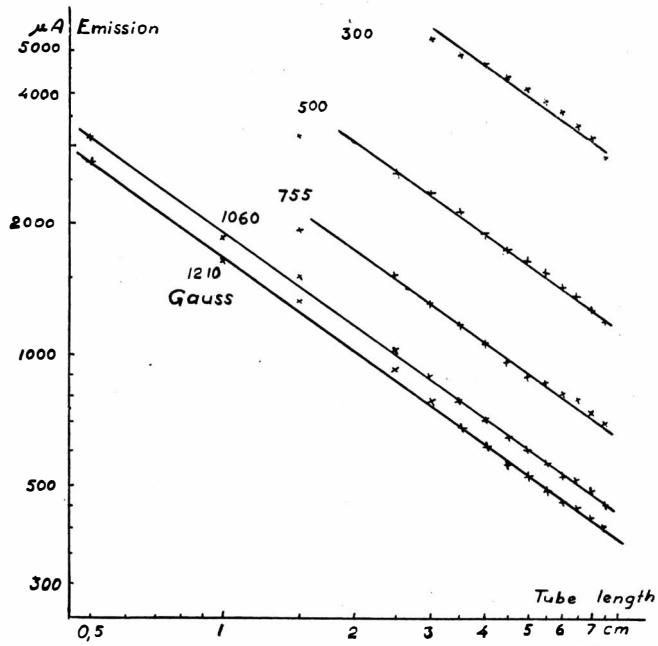

Fig. 8. The relation between the emission and tube length for a constant value of the ratio of rail current to emission.

Tube J15
Anode voltage 400 Volts
Cathode voltage 50 Volts

$$\frac{\text{Rail current}}{\text{Emission}} = 0.001$$

As the space charge density decreases the electrons must travel a longer way before a certain number of them have gained sufficient energy to be able to reach a negative electrode making, therefore, a larger emission necessary to give a certain arbitrary percentage N-current in a short tube, than is necessary in a long one. From a practical point of view this means that the longer the trochotrons are made, the smaller will be the emission permissible, since the percentage N-current must be maximized.

d) The similarity principle

Measurements were made on three similar tubes, having their dimensions proportional to 1:4:16 (cp. the tube table). In the smallest of these, the breadth of which was only 1.9 mm, it was difficult to get a satisfactory construction, the ends of the filament

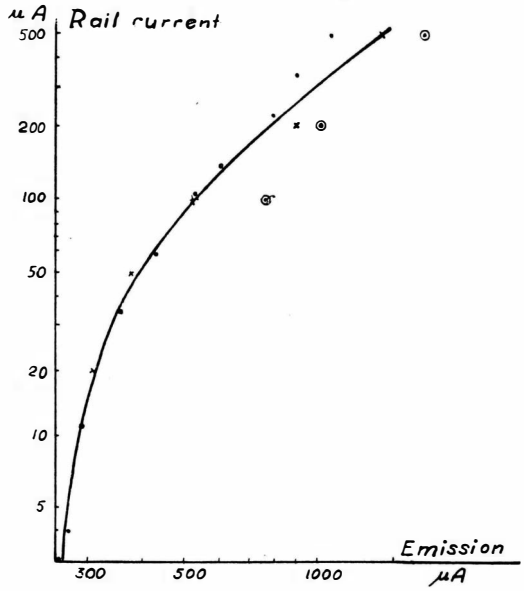

Fig. 9. The relation between rail current and emission for three similar tubes.

	Anode voltage 200 Volts	
	Cathode voltage	10 Volts
. .	Tube J8: 1;	B = 150 Gauss
× ×	Tube J8: 3;	B = 600 Gauss
⊙⊙	Tube J8: 5;	B = 2 400 Gauss

being outside the side plates, this meaning that a considerable part of the emission travelled to the receiving plate between the electrodes and the bulb. Of interest is the current within the tube, and this is somewhat smaller than the emission determined in the experiment. Although here an error is introduced, it is possible to reach some valuable conclusions.

During the experiment, the voltage was the same for all three tubes, but the magnetic field strength was inversely proportional to the size of the tubes. The N-current was measured as a function of the emission, see figure 9. (For discussion see 7.)

e) N-current as a function of the cathode voltage

Now we keep the anode voltage, the magnetic field strength and the emission constant, and determine the N-current as a function of the cathode voltage. In figure 10 we have plotted the ratio of N-cur-

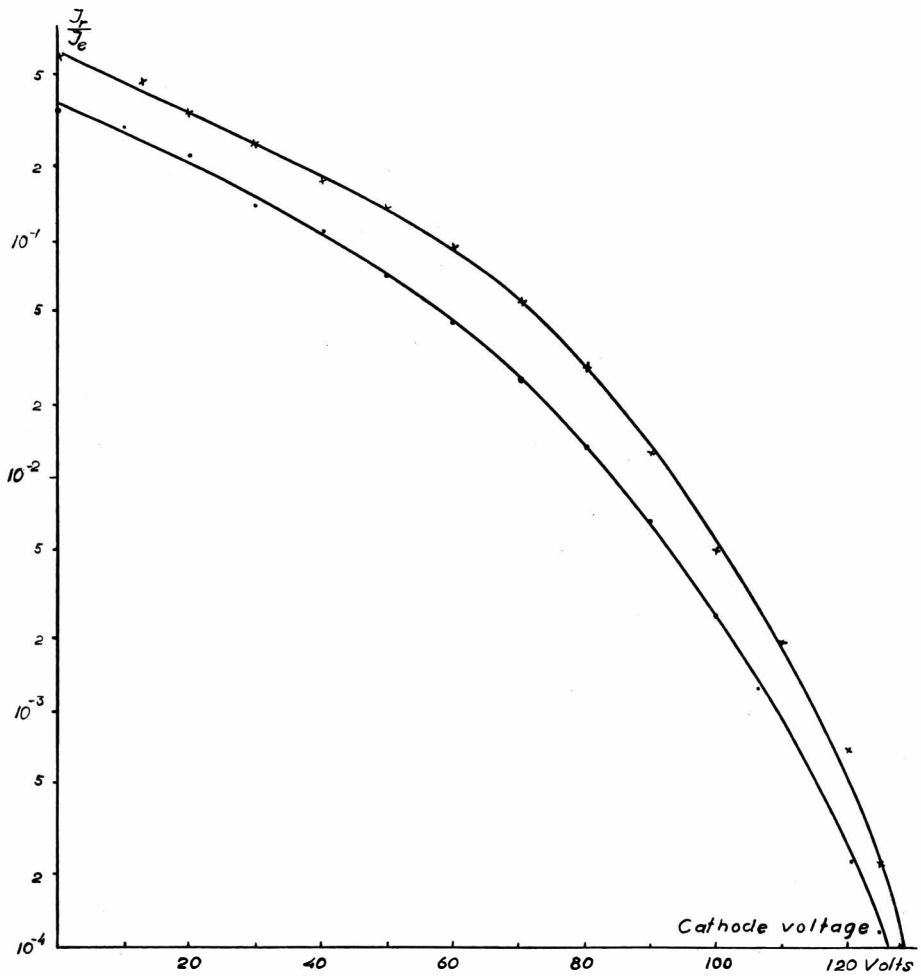

Fig. 10. The ratio of N-current to emission as a function of cathode voltage.
Tube J15
Anode voltage 400 Volts
×× B = 1 130 Gauss; Emission = 1 000 μA
⊙⊙ B = 750 Gauss; Emission = 1 300 μA

rent to emission versus the cathode voltage and we can see that the N-current decreases with increasing cathode voltage.

f) Dependence on the anode voltage

Here we keep the magnetic field strength and the ratio of the cathode voltage to the anode voltage constant, and determine the

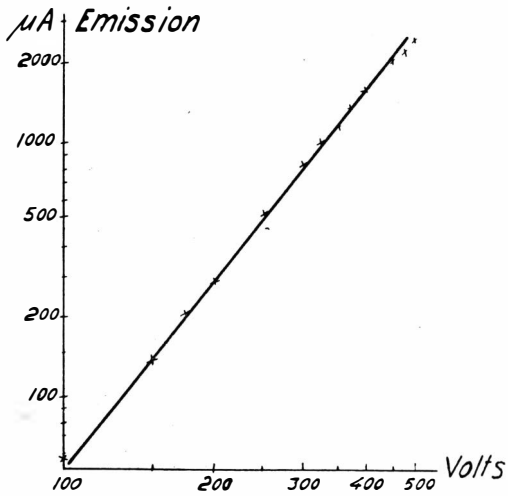

Fig. 11 a. Relation between emission and anode voltage for constant ratio of rail current to emission (0.05).
Tube J2
Cathode voltage 0 Volts
Magnetic induction 200 Gauss

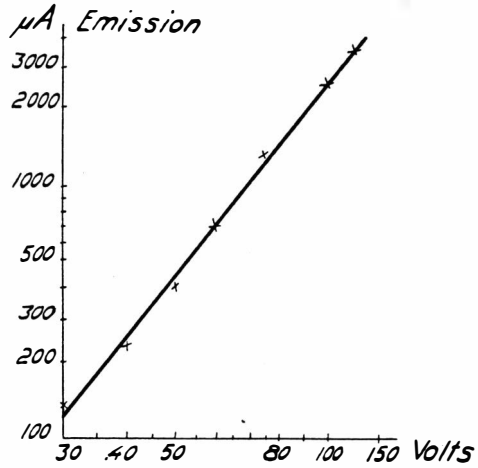

Fig. 11 b. Relation between emission and anode voltage for constant ratio of rail current to emission (0.05).
Tube J5
Cathode voltage 0 Volts
Magnetic induction 250 Gauss

value of the emission that gives a certain arbitrary value (5 %) of the ratio of rail current to emission, and repeat this procedure for different anode voltages.

In figure 11 a and b we can see that the emission which gives a certain percentage N-current increases with the anode voltage.

These two curves refer to two tubes, one of which has about five times the breadth of the other. Further, the first one has an oxide cathode and the other a high temperature cathode. Both these curves, however, have the same character.

g) Dependence on the magnetic field strength

All voltages are kept constant. The emission that gives a certain arbitrary percentage N-current (5 %) is determined, and this procedure is repeated for other magnetic field strengths. In figure 12 the emission is plotted versus the magnetic field strength for the two above-mentioned tubes.

It is evident that the higher the magnetic field strength, the lower is the permissible emission for a certain percentage N-current.

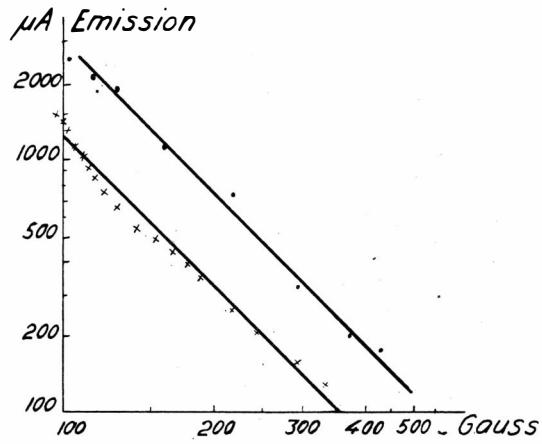

Fig. 12. Relation between emission and magnetic field strength for constant ratio of rail current to emission (0.05).

×××	Tube J2	. . .	Tube J5
Anode voltage	200 Volts	Anode voltage	50 Volts
Cathode voltage	0 Volts	Cathode voltage	0 Volts

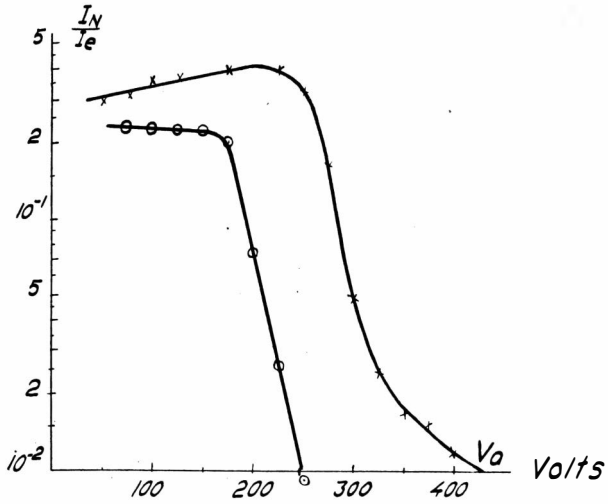

Fig. 13. N-current to the rail and to the side plates as a function of the anode voltage.

Tube J9
Cathode voltage 0 Volts
Magnetic induction 175 Gauss
Emission 590 μA at 300 Volts anode voltage
$\times\times\times$ N-current to the rail
$\odot\odot\odot$ N-current to the side plates

h) Distribution of N-current between the rail and the side plates

In this particular case we used a tube which did not have the rail and side plates connected. Hence we could measure the current to each of them separately.

The magnetic field strength and the cathode temperature were kept constant, and the cathode voltage was zero. The relative N-current to the side plates (connected together) and to the rail are in figure 13 plotted versus the anode voltage.

When the anode voltage is low the emission has been found to be space charge limited, and there the ratio of N-current to emission is nearly constant, as shown in figure 13. Above a certain voltage the mentioned ratio rapidly decreases and in this region the emission has been found to be nearly constant.

Figure 13 shows that in the temperature limited region the predominant part of the N-current goes to the rail, and only a tenth

to the side plates, but in the space charge limited region the difference is very much smaller.

6. *An empirical formula for the N-current*

The most concise abstract of the measurement made is the empirical formula

$$I_r = I_e \cdot e^{-X}$$
$$X = f\left(\frac{V_c}{V_a}\right) \cdot \frac{V_a^{3.7}}{I_e^{1,5} \cdot B^3 \cdot l} \qquad (1)$$

I_r = N-current to the rail — V_a = Anode voltage
I_e = Emission — V_c = Cathode voltage
l = Tube length — B = Magnetic induction

When I_r/I_e is very small, an exponential function, a parabola of higher degree and e. g. (1) can illustrate the observed dependance on emission equally well. Since of the mentioned formulae only (1) is satisfactory, also for large values on I_r/I_e this is prefered here.

From figure 10 we can conclude that for low values of the cathode voltage I_r/I_e is approximately an exponential function of the cathode voltage i. e. f in (1) is a linear function. At higher values of the cathode voltage, however, a term of second power must be added, but then good agreement with the observed points will be obtained.

Figure 11, 12 and 8 show that the relations between I_e and V_a, I_e and B, I_e and 1 for I_r/I_e = constant can be described by straight lines in log scales. These measurements can be summarized in the formula

$$I_e \cdot B^2 \cdot V^{-2,5} \cdot l^{0,7} = \text{const. for } I_r/I_e = \text{const.} \qquad (2)$$

For discussion see LINDBERG.[1])

From figure 8 we can conclude that for I_r/I_e = constant is $I \cdot l^{0,7}$ = constant a good approximation (*cp.* (2)). The longer the tube, the smaller the permissible emission.

The derivative of (1) with respect to l has the same character as the curves in figure 3 and 4 which are obtained experimentally. The real increase with distance from the cathode (figure 6) resp.

[1]) LINDBERG, L. Design of Trochotron Circuits.

with emission (figure 5), however, is much larger than that given by (1) before the maximum is reached.

In trochotron tubes used in practical applications the emission is usually kept constant and the beam is directed to electrodes at different distance from the cathode.[1]) The more remotely an electrode is situated from the cathode, the fainter is the beam current (part of the emission has gone to the rail) and the more it has been widened (in the direction of the electric field). Consequently the different electrodes must have different properties. This has also been confirmed experimentally. Only for large emission is the difference considerable.

7. *Discussion of the similarity principle*

The curves for the two larger tubes in figure 9 coincide quite well, and the curve for the smallast tube almost coincides with the two other curves. According to the present measurements the emission is 700 and 1 000 μA for the largest resp. smallest tube (figure 9) for a given value on the N-current.

For constant emission the increase in size 1: 16 causes an increase in the space charge density 1: 256. Since such a large change in the space charge density and a simultaneous increase 16 times in the magnetic field strength does not alter the emission more than 30 %, for a constant N-current, one must conclude that if the electric and magnetic field strengths are varied inversely proportional to the size of the tube, but the emission is kept constant, the N-current will remain constant.

If we can tolerate a certain amount of N-current and, as is often the case, the anode voltage at our disposal is limited, the emission must be below a certain value, often smaller than desirable. By decreasing the size, no improvement in this respect can be expected, since, according to the similarity principle, unchanged emission gives unaltered N-current. Only if the smaller interelectrode capacitancies (or the smaller size of the tube) are valuable, is the decrease of the tube size of any interest.

If, however, after a similar decrease of the tube, only the breadth is restored to its former value, it would seem possible that the current can be increased, in the same proportion as the increase in the breadth.

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[1]) ALFVÉN, H. On Trochoidal Electronic Beams and Their Use in Electronic Tubes (»Trochotrons»).

II. Noise in the trochotron

In the trochotron as much emission is wanted as possible. Long before the emission is space charge limited, a severe noise, which rapidly increases with emission, appears, and the noise level grows much larger than that corresponding to ordinary shot noise. Since the applications often determine an upper limit for the tolerable noise level, that level then determines the largest emission to be used. Study of this noise is consequently of great interest.

1. *The noise as a function of emission*

The anode and receiving plate were at positive potential, and the latter electrode received the whole emission. The receiving plate was loaded by a 100 ohms resistor and the noise appearing across it was indicated by an amplifier (Hallicrafters *SX* 42) with a linear detector. To insure that no a. c. voltage occurred at any electrode except the receiving plate, they were connected the shortest way to the rail through capacitors (10 000 $\mu\mu F$, mica).

The noise was compared with that from a saturated diode (an 812 tube with the grid connected to the anode). The anode of this tube was connected to the receiving plate and its cathode to the rail.

With only the trochotron filament heated, a noise was observed. The diode current necessary to obtain the same noise current, with the trochotron inoperative, can then be taken as a measure of the trochotron noise, since in both cases the noise spectrum is flat within the regarded, small frequency band.

The square root of the ratio of diode current to trochotron emission must then be equal to the ratio of experienced noise to the shot noise, corresponding to the trochotron emission. In figure 14 the former ratio is plotted versus the trochotron emission. We can see that up to about 1 500 μA a noise of the shot type appears but that above this value the trochotron noise increases much more rapidly with emission than does the shot noise.

This measurement indicates that in the trochotron two types of noise appear; first the ordinary shot noise, and secondly another type of noise which we can call »N-noise» since it probably has the same cause as the N-current.

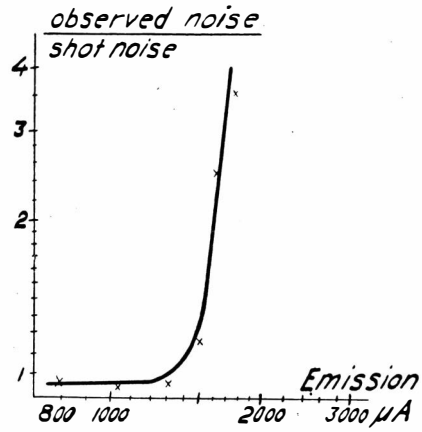

Fig. 14. Ratio of appearing noise to shot noise level at 3 Mc/s as a function of emission.

Tube J2	
Anode voltage	1 000 Volts
Cathode voltage	100 Volts
Magnetic induction	250 Gauss

The first one increases proportionally to the square root of the emission but the latter type at a much higher power, and therefore the N-noise can be ignored (relative to the shot noise) at a low value of emission, and vice versa, at high emission.

Since the N-noise increases so rapidly with the emission we soon reach a diode current, for the same noise power as in the trochotron, which can hardly be realized. In this region the diode is replaced by a standard signal generator.

At the detector after the amplifier the trochotron noise gives a certain voltage. With the trochotron replaced by the generator, the output of the latter was adjusted to give the same detector voltage, as was obtained when the trochotron was coupled to the amplifier. The noise voltage from the trochotron must then be proportional to the sine wave voltage from the generator.

The noise was measured as a function of emission for some values of anode voltage and the result is given in figure 15. In this region also, the noise increases much more rapidly than does the ordinary shot noise.

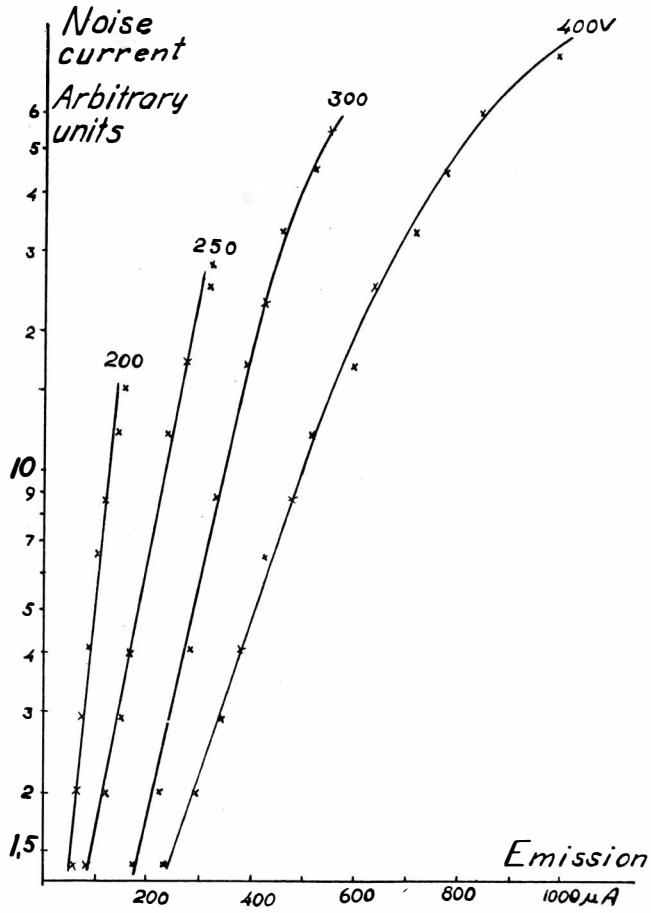

Fig. 15. The noise at the receiving plate as a function of emission with the anode voltage as a parameter.
Tube J2
Cathode voltage = 0.1 × anode voltage
Magnetic induction 300 Gauss

2. *Relation between the noise and the anode voltage for a constant noise level.*

The magnetic field strength was kept constant. The emission which gave noise of a certain arbitrary level was determined and the measurement was repeated for several anode voltages.

The curve obtained in figure 16 shows that the necessary emission for a constant noise level increases with the anode voltage for low

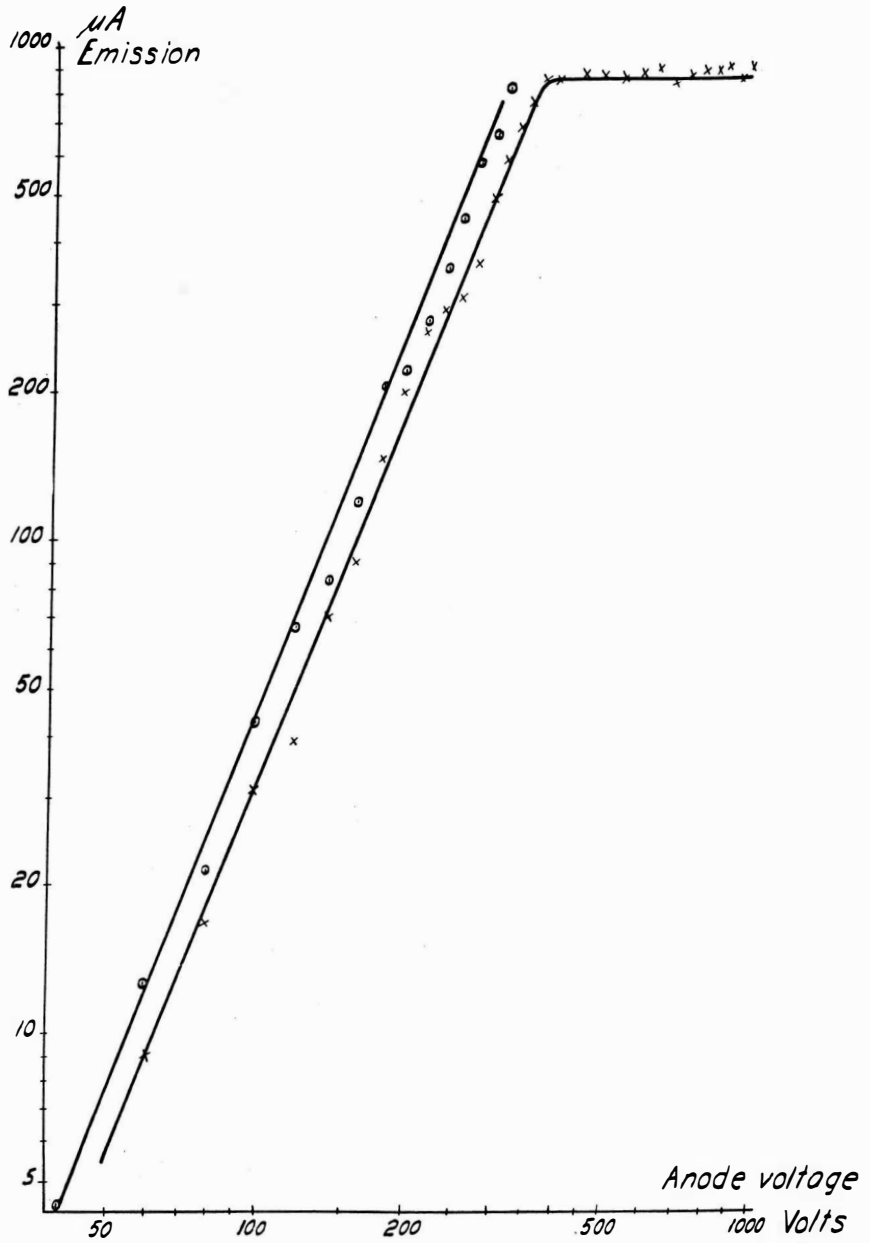

Fig. 16. Relation between anode voltage and emission for constant noise level.
Tube J18
Cathode voltage = 0.1 × anode voltage
Magnetic induction ×× 936 Gauss
⊙⊙ 815 Gauss

anode voltages but is independent of this voltage when it exceeds a certain value. The transition between these regions seems to be quite sharp.

In the steep part of the curve the N-current dominates, and consequently we can conclude that the necessary emission for a given arbitrary N-noise level increases with the anode voltage. In the right part of this figure, however, the N-noise can be ignored, relative to the shot noise. Since the shot noise is independent of the anode voltage, the slope must be zero.

3. *Relation between the emission and the magnetic field strength for a constant noise level*

In this case the anode voltage was kept constant and the emission was increased to give a certain arbitrary noise level, and this procedure was repeated for several values of the magnetic field strength. The result can be found in figure 17.

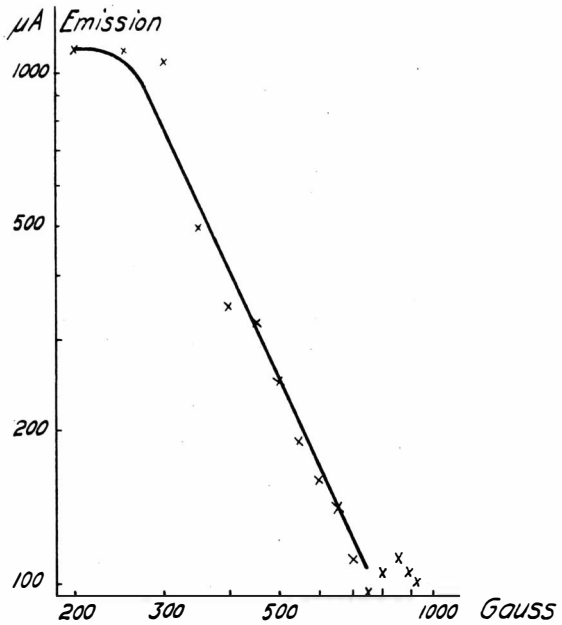

Fig. 17. Relation between the magnetic field strength and the emission for constant noise level.

Tube J2	
Anode voltage	1 000 Volts
Cathode voltage	100 Volts

Here also it is possible to distinguish two distinct regions, one at low magnetic field strength, where the emission is independent of the field strength, and another at high magnetic field strength with rapidly decreasing emission.

In the latter region we have the dependance between the emission and the field strength for the N-noise level constant (shot noise negligible), and vice versa for the former region. We can thus conclude that for constant N-noise level the necessary emission decreases with increasing magnetic field strength.

4. *The frequency spectrum of the noise*

The principal purpose of this measurement was to find out whether anything particular happens at the angular frequency $\frac{eB}{m}$. To ensure that this frequency falls in a region where measurements are relatively easy to perform, a small magnetic field strength is necessary and hence, in accordance with the similarity principle (which is probably also valid for the noise) a large tube (tube J 8: 1 in the tube table).

The receiving plate was on rail potential so that the electrons, after having passed the tube, went to the anode. The noise voltage was measured over a 100 ohms resistor in the anode lead. To make the load reactance zero, a parallel resonant circuit, which, with the tube reactances included, was tuned to the frequency to be measured was connected across the 100 ohms resistor.

The noise voltage was measured by an amplifier, having a square law detector.

For calibration a standard signal generator with an internal resistance of 100 ohms was connected to the amplifier instead of the trochotron. The sine wave voltage from the generator which gave the same voltage at the detector as the trochotron was determined. If the bandwidth of the amplifier is then known, the noise current from the trochotron can be computed.

The common and more accurate method, by comparison with a diode cannot be applied here, since diode currents of too great a magnitude would have been necessary.

The spectrum from 0.5 to 130 *Mc/s* was examined. In figure 18 the results for three values of the magnetic field strength are reproduced. All these three curves have the same character. In one

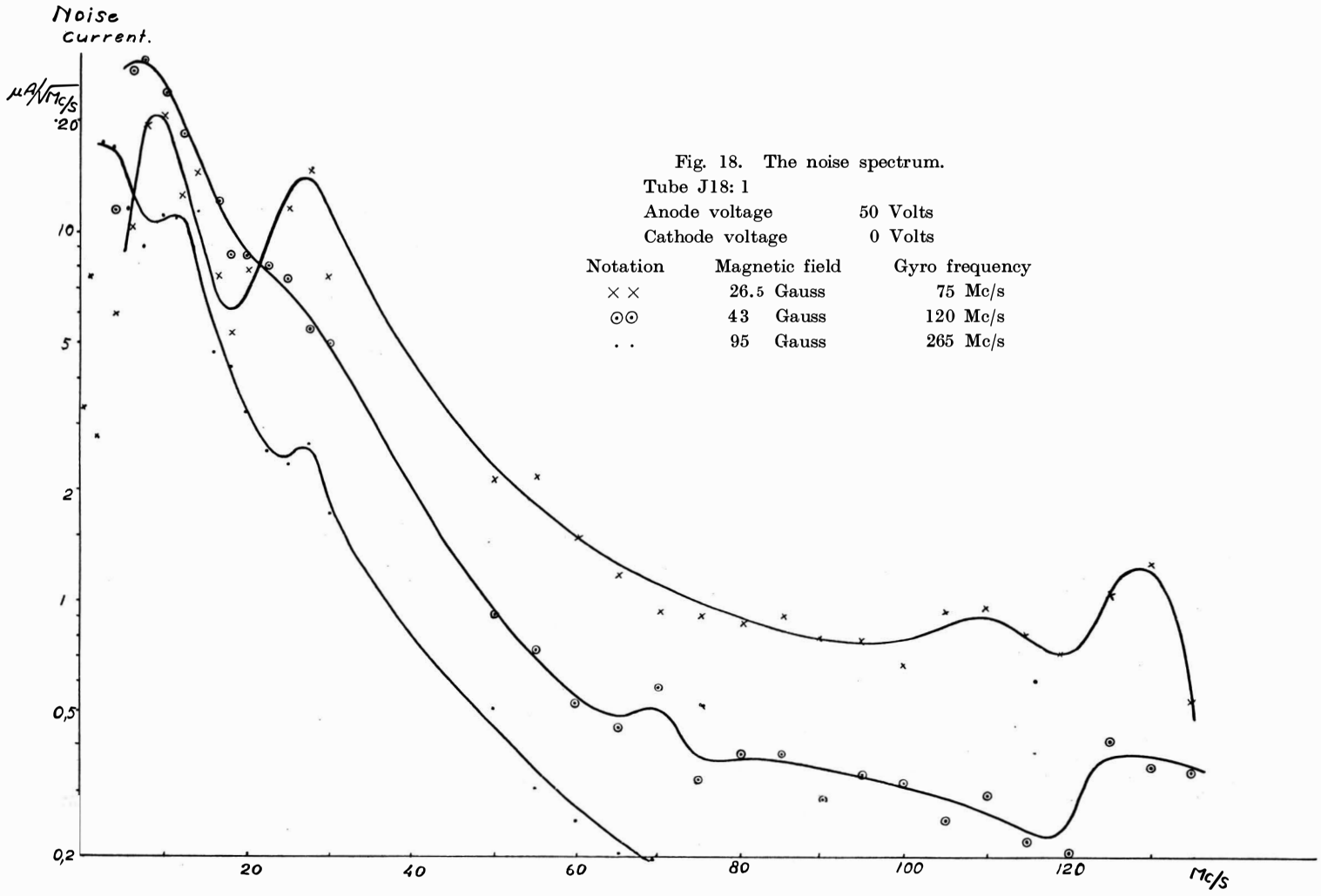

Fig. 18. The noise spectrum.

Tube J18: 1

Anode voltage 50 Volts

Cathode voltage 0 Volts

Notation	Magnetic field	Gyro frequency
× ×	26.5 Gauss	75 Mc/s
⊙⊙	43 Gauss	120 Mc/s
. .	95 Gauss	265 Mc/s

of them the gyro frequency was 75 Mc/s but nothing remarkable can be found at this frequency. The two other curves, having higher gyro frequencies, show the same behaviour at 75 Mc/s as the former ones, and therefore we must conclude that nothing particular happens at the gyro frequency.

5. *Discussion*

Within the small region presented in figure 15, an exponential function seems to be as good an approximation as a parabola of higher degree. In the latter case, the slope in a log scale seems to be independant of the values on the electric and magnetic field strengths. The noise is proportional to I_e^m where m is approximately 2.8.

Figure 16 shows that for constant N-noise level the relation between the emission and the anode voltage is a straight line in the log scale, giving $I_e \cdot V_a^{-2,5} =$ constant. Here we obtained exactly the same relation as for the corresponding N-current. This probably indicates that the N-current and the N-noise have the same cause.

The preliminary measurement in figure 17 indicates that for constant noise level $I_e \cdot B^n =$ constant where $n \approx 2.2$, which is fairly near the corresponding value for the N-current case. It is possible that a more accurate measurement here also will give the value 2.0 to the exponent as in the N-current case.

In hot cathode arcs an intense noise appears and in measurements made by Cobine and Curry[1]) we can observe that at 1 Mc/s the noise current for a 6 $D4$ tube is about 500 $\mu A/\sqrt{Mc/s}$ when $B = 375$ Gauss and the anode current 5 mA.

When reducing the size of the trochotron tube used for the spectrum measurement to the same size as the $6D4$ tube, the magnetic field strength increases to about the same value as that used in the $6D4$ case. Then both the noise level and the emission are approximately 1/20 of the values in the measurements of the gaseous discharge. By increasing the emission in the trochotron only about three times the same noise level as for the $6D4$ tube will probably be obtained. Thus, in both these cases, a noise of about the same magnitude is obtained.

The greatest difference is, then, that the noise spectrum in the

---

[1]) Cobine, J. D., and Curry, J. R. Electrical Noise Generators. Proc. Inst. Rad. Eng. vol 35 p 875 1947.

high vacuum case is extended to much higher frequencies than in the gaseous discharge.

Since, approximately, the same noise level seems to be obtainable if gas is present or not, other conditions remaining unaltered, it is possible that the same mechanism causes the noise in both cases.

Acknowledgement

My sincere thanks are due to Professor H. ALFVÉN for the interest he has shown in the present work and for his valuable comments on it.

III. The most important properties of the tubes used

J2 Size of electrode structure: Breadth × height × length 15 × 10 × 60 mm. Oxide coated nickel band as filament. The leads to the electrodes shortest way through the glass bulb.

J5 70 × 11 × 55 mm. Cathode: 0.12 mm tungsten wire.

J8: 1 30 × 25 × 165 mm. Oxide coated, thin nickel wire.

J8: 3 7.5 × 6 × 41 mm. Oxide coated, thin nickel wire.

J8: 5 1.9 × 1.5 × 10 mm. Oxide coated, thin nickel wire as cathode.

J9 The same as J2 but with the side plates not connected to the rail.

J15 20 × 10 × 85 mm. Filament: Oxide coated nickel band. The rail divided in segments.

J18 15 × 5 × 100 mm. Cathode: 0.12 mm tungsten wire. Electrode leads direct through bulb. (Special care was taken to prevent electrons from escaping to the space between the electrodes and the bulb.)

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ON THE INSTABILITY OF AN ELECTRON GAS IN A MAGNETIC FIELD

BY

K. G. MALMFORS

During the development of the trochotron tubes it has been observed that — under certain conditions — a large part of the electron current goes to electrodes which are negative with respect to the cathode. Similar effects have been observed in cylindrical magnetrons. The effect has been subjected to a thorough experimental study, the details of which have been published in the preceding section of this journal.[1]) In this paper some theoretical aspects of the process will be proposed and the conclusions will be compared with the experimental results.

The effect has been studied in tubes of the type shown in Fig. 1. In the combined electric and magnetic fields the electrons move in trochoidal paths in a direction perpendicular to the electric and the magnetic field. The potential distribution in the tube is such that the electrons cannot move unaffected in the direction of the magnetic field but are limited to oscillating through the symmetry plane. When the rail and the side plates are given a potential negative with respect to the cathode it seems — according to ordinary laws of motion — impossible that electrons should go from the cathode to these electrodes. Experiments show, however, that quite a considerable part of the emission is collected by this negative electrode. The effect cannot be understood unless energy has been exchanged between the electrons. The energy gain is of such magnitude that it cannot be explained as being due to random accelerations in the electron gas.[2])

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[1]) E. ÅSTRÖM: Experimental Investigation on an Electron Gas in a Magnetic Field.

[2]) An electron which goes L cm through an electron gas, containing n electrons per cm^3 will on the average change its energy by the amount $\Delta W = e^2 \sqrt{n \cdot L}$. In the present case ($n \simeq 10^9$; $L \simeq 10$ cm) $\Delta W \simeq 0.01\ eV$.

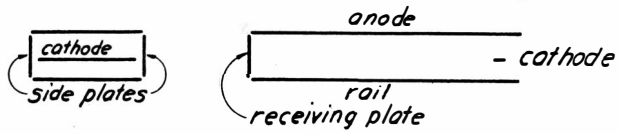

Fig. 1. Experimental tube. The magnetic field is parallel to the cathode.

For a study of the conditions in the tubes it is convenient to regard the electrons from a co-ordinate system which moves with the drift velocity $c \cdot \frac{E}{H}$. In this system the electric field vanishes and the electrons move in circular orbits. Furthermore, let us disregard the oscillation along the magnetic field and consider a system where isotropically distributed electrons move in circular orbits on planes which are perpendicular to a homogeneous magnetic field. Since the time of revolution is the same for all electrons $\left(\tau = \frac{2\pi mc}{eH}\right)$ any distortion of the distribution of the electrons will be repeated with the period τ. Such a periodic variation of the charge density gives rise to electric fields varying with the same period. As one electron will repeatedly pass the same »accelerating gap» — analogous to what is the case in the cyclotron — it can achieve an energy change which is appreciably larger than if the accelerations were distributed at random. If the density fluctuations in the gas are due to the statistical structure, even such »systematic» accelerations will produce an effect which is small compared with the observed one. (In this case the effect is increased by a factor $\sqrt{N}$, where N is the number of revolutions. Since, in the experiment tubes, N is of the order 10^2—10^3, the effect cannot account for the great discrepancy.)

The main influence of a magnetic field acting upon an electron gas is that density fluctuations will not immediately be smoothed out — as would be the case if the magnetic field were not present — but are repeated with the period τ. As regards the stability of the electron gas there are two alternatives. On the one hand the amplitude of the fluctuation may successively decrease. On the other hand, there seems to be no principal reason excluding the possibility that a system is unstable i. e. such that a primary distortion gives rise to a larger one after the period τ which then is further amplified during the next period and so on. The following discussion is

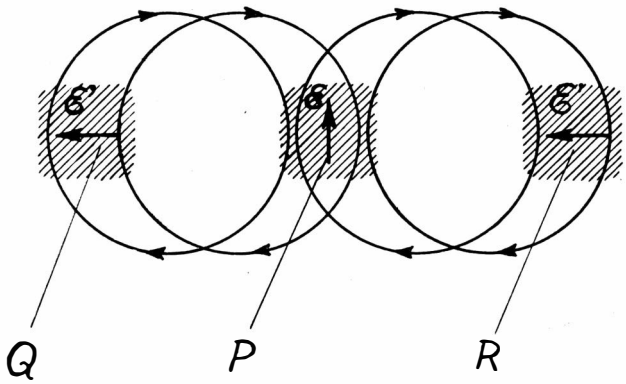

Fig. 2. Distortions in the electron gas.

intended to show that the latter alternative may be the case under certain conditions.

Let us consider the same system of electrons as above, and furthermore assume that all electrons have about the same radius of curvature, ϱ, in the magnetic field. As an initial condition we assume that the uniform charge density will be disturbed in such a manner that an electric field, $\mathcal{E}$, arises within a region P, the dimensions of which are some number of times less than ϱ. This electric field will exist during a time which is less than τ in the same proportion. Electrons which pass this region obtain a change of momentum

$$\Delta p = e \cdot \mathcal{E} \cdot \alpha \cdot \tau \tag{1}$$

where

$$\tau = \frac{2 \pi mc}{e H} \tag{2}$$

α being a numerical factor, some times less than unity. Geometrically it can be shown that a momentum change Δp is equivalent to a displacement of the circle's centre.

$$\Delta s = \frac{c}{e H} \cdot \Delta p \tag{3}$$

Δs being perpendicular to Δp (when the components perpendicular to H are considered). This means that — as a consequence of the displacement of the electrons — there arises an electric field in the

regions Q and R (c. f. Fig. 2) half a period after the primary distortion. If $\mathfrak{E}'$ denotes the strength of this new electric field there applies:

$$\mathfrak{E}' = \beta \cdot 4\pi \cdot n\, e \cdot 2\, \Delta s \tag{4}$$

where n is the electron density and β is a numerical factor which is less than unity and denotes that fraction of the electrons in the regions Q and R which have passed the region P. By combining (1), (2), (3) and (4) one obtains:

$$\mathfrak{E}' = \alpha \cdot \beta \cdot (4\pi)^2 \cdot m\, c^2 \cdot \frac{n}{H^2} \cdot \mathfrak{E} \tag{5}$$

Formula (5) shows that the stability of an electron gas is closely connected with the magnitude of the expression $\alpha \cdot \beta \cdot (4\pi)^2\, m\, c^2 \cdot \frac{n}{H^2}$.

It may be of interest to point out here that an expression of the same form is obtained when the frequency of plasma oscillations, $\omega_{pl} = \sqrt{\frac{4\pi n e}{m}}$, is divided by the gyromagnetic frequency, $\omega_g = \frac{e H}{m c}$,

$$\left(\frac{\omega_{pl}}{\omega_g}\right)^2 = 4\pi\, m\, c^2 \cdot \frac{n}{H^2}$$

The increase of a primary distortion in the lapse of time can be estimated by means of formula (5), which will now be written

$$\mathfrak{E}' = k \cdot \mathfrak{E} \tag{5'}$$

Fig. 3 gives a schematic picture of the increase and the spread of a primary distortion during four half periods. Such a simple consideration shows that the induced electric fields have a tendency to *amplify* the primary distortion.

As far as the system of Fig. 3 is considered the primary vector increases with time as $1 + \frac{1}{6} k^4 \cdot N\,(N+1) \cdot [N\,(N+1) - 2] + \ldots$

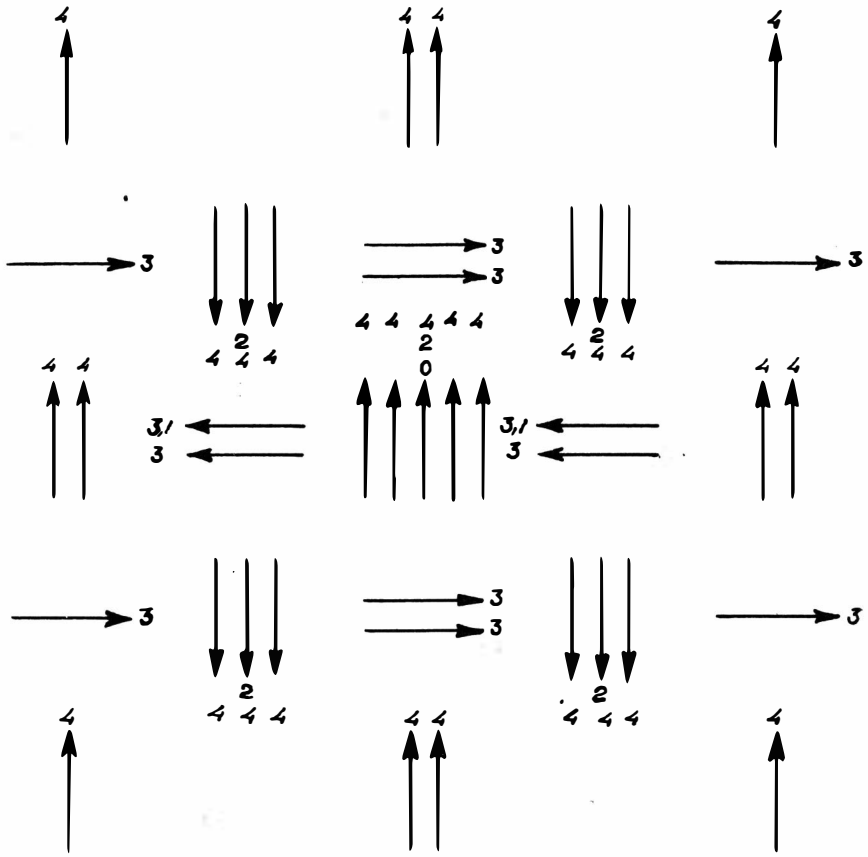

Fig. 3. Increase of a distortion in the lapse of time. The indices at the arrows indicate the number of half periods after the primary distortion.

where N denotes the number of revolutions, $N = \frac{t}{\tau}$. When $\frac{k\,t}{\tau}$ approaches unity, the induced vectors will be of the same order of magnitude as the primary one and the following formula will hold approximately:

$$\mathfrak{E}_{t+\tau} = \mathfrak{E}_t + 2\,k\,\mathfrak{E}_{t+\frac{\tau}{2}} \tag{6}$$

which gives:

$$\mathfrak{E}_t \sim e^{\frac{2\,k\,t}{\tau}} \tag{7}$$

If the sum of all distortions $\int |\mathcal{E}|\, dv$ at the time t is denoted by s_t, there applies:

$$s_{t+\tau} = s_t + 2\, k\, s_{t+\frac{\tau}{2}}$$

and:

$$s_t \sim e^{\frac{2\,k\,t}{\tau}}$$

Since $\frac{1}{8\,\pi} \int \mathcal{E}^2\, dv$ gives a measure of the energy of the electric fields this expression illustrates, probably more adequately, the growth of the distortion. If S_t denotes the quadratic sum of all distortions there applies:

$$S_t \sim e^{\frac{4\,k\,t}{\tau}}$$

From this discussion it may be concluded that distortions in the electron gas increase exponentially with time, the exponent involving the expression $\frac{k\ t}{\tau}$.

This discussion referred to a system of electrons which was restricted by some particular assumptions. As we shall see, these assumptions seem to be justified as regards the conditions in the experimental tubes.

1:o. No error arises from the fact that the drift velocity has been neglected, but it must be remembered that the number of revolutions is limited by the time of travel in the tube.

2:o. It has been assumed that the electron gas is not limited in the two dimensions perpendicular to the magnetic field. In the case of the experiments the one dimension is limited, since the electron beam is rather thin in the direction of the electric field. Even if the thickness of the beam is only some few radii of curvature, the above-mentioned mechanism is still applicable, the only difference being that the exponent has to be divided by some factor between 1 and 2.

3:o. Formula (4) contains n, the electron density, which has been assumed to be a constant in the whole volume (exept for the distortions, which are small). In the tubes n is a function of the

distance from the symmetry plane. No great mistake is made, however, if a region about the symmetry plane is considered, within which the density varies some few times.

4:o. The oscillation along the magnetic field has been neglected. The deduction is still valid, in the above-mentioned region, if one more constant, $1 - \delta$, is inserted in formula (5), δ denoting that fraction of the electrons which leaves the region during the time taken by half a revolution. Since this time is negligible compared with the time of oscillation in the direction of the magnetic field, the correction is of little importance.

5:o. The assumption that all electrons have about the same radius of curvature seems to be of essential importance for the whole mechanism. In the tubes the radius of curvature is determined by the electric and magnetic fields and thus it would seem reasonable that this assumption also is fairly satisfactory.

6:o. Finally, it should be observed that the discussion is valid only for small distortions.

Comparison with the experiments. One of the first results as regards the occurence of current to negative electrodes (N-current) in the trochotrons was that the current could be observed only when the emission, i_e, exceeded a certain value, which, furthermore, depended upon the applied electric and magnetic fields. It could be shown that the phenomenon is governed by the magnitude of the expression

$$\frac{i_e H^2}{E^{2,5}} \tag{8}$$

The dependence upon the length of the tube was investigated by means of a tube with the negative electrode split up into several equidistant parts. By keeping H and E constant in this tube it could be shown that — approximately — the emission requested is inversely proportional to the length of the tube. Thus the above-mentioned expression has to be completed as follows:

$$\frac{i_e H^2 l}{E^{2,5}} \tag{8'}$$

Furthermore, it could be shown that the N-current varies exponentially with the emission. Thus the general expression for the N-current takes the form:

$$i_N \sim e^{C \cdot \frac{i_e H^2 l}{E^{2,5}}} \tag{9}$$

The expression is applicable in the region where $i_N \ll i_e$.

The mutual dependence of the parameters as expressed in (8) has also been verified by measuring the noise at the end electrode and it has been shown that also this noise increases exponentially with the emission. Thus an expression of the form (9) is valid also for the noise in the tubes. It seems possible that the expression for the field energy, $\frac{1}{8\pi} \cdot \int \mathfrak{E}^2 \, dv$, calculated above, is a direct measure of the noise energy. If this be the case, the theoretically calculated exponent $\frac{k t}{\tau}$ should be compared with the experimental one. By applying formulae (2) and (5) and by considering that $t = \frac{l}{c \frac{E}{H}}$ one obtains:

$$\frac{k t}{\tau} = 8 \pi \alpha \beta \frac{e n l}{E}$$

If Δ is the cross-section of the electron beam there applies:

$$e \cdot n = \frac{i_e}{\Delta \cdot c \frac{E}{H}}$$

Thus:

$$\frac{k t}{\tau} = \frac{8 \pi \alpha \beta}{c} \frac{i_e H l}{E^2 \Delta} \tag{10}$$

The cross-section of the beam has not yet been determined experimentally and therefore Δ can only be estimated. It seems reasonable that Δ is a function of the radius of curvature which is given by the formula

$$\varrho = \frac{m c}{e} \frac{E}{H^2}$$

Since Δ probably increases with ϱ, formula (10) gives an expression, which has the same character as (8′).

Complete agreement would be achieved if Δ increases proportionally to the square root of ϱ.

Furthermore, the exponent calculated theoretically is of the same order of magnitude as the experimental one. From one experiment series[1]) there results that the exponent may be written 30 $i_{m\Delta}$ when $l = 6$ cm, $H = 300$ Gauss and $E = 400$ V/cm. With the known figures inserted in formula (10) $\frac{kt}{\tau}$ takes the form: $10 \frac{\alpha\beta}{\Delta} i_{m\Delta}$. This means that there is quantitative agreement if $\frac{\alpha\beta}{\Delta}$ has a value about unity, which is obtained with reasonable values of α, β, and Δ. ($\alpha = 0.2$; $\beta = 0.2$; $\Delta = 0.05 \times 0.5$ cm^2;)

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[1]) Cf. Fig. 15 loc. cit.

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